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COMPLEX LOADING AND STABILITY OF SHELLS MADE OF POLYMERIC MATER--ETC(U)

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COMPLEX LOADING AND STABILITY OF SHELLS MADE OF POLYMERIC MATERIALS (CHAPTER V)

by

G. A. Teters

FOREIGN TECHNOLOGY DIVISION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh^{-1}
cos	cos	ch	cosh	arc ch	cosh^{-1}
tg	tan	th	tanh	arc th	tanh^{-1}
ctg	cot	cth	coth	arc cth	coth^{-1}
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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Chapter V.

STABILITY OF PLATES AND SHELLS MADE OF POLYMERIC MATERIALS WITH CONSTANT LOAD.

§5.1. Introductory observations, the prerequisite/premises of investigation.

Many structural cell/elements from polymeric materials long time are located under the effect of external loads. In view of the fact that polymeric materials possess the properties of creep, under the effect of external load they transform in time and arises the question concerning the prolonged stability of constructions made of these materials. The studies of prolonged stability preceded works on the stability of glass-plastic shells with intermittent load.

Of the special feature/peculiarity of the behavior of shell, made from glass-fiber-reinforced plastic, under load in essence are caused by the properties of material.

Glass-fiber-reinforced plastic possesses a whole series of special feature/peculiarities, in particular: by the high specific strength in the direction of reinforcement, by the relatively low moduli of elasticity, pronounced heterogeneity, the possibility of the optimum construction of material, etc. In connection with the low moduli of elasticity of the material of experiment in the stability of glass-fiber-reinforced plastics, acquires the special importance.

Glass-fiber-reinforced plastic is the composite laminate, which consists of two components: by almost elastic, by the determined form of the oriented fittings (glass filaments, glass cloth, etc.) and of isotropic polymer - bonding agent, that possesses the properties of viscoelasticity. Thus, during the construction of the theory of the deformation of shell of glass-fiber-reinforced plastic it is necessary to examine the questions, connected with heterogeneity, anisotropy and the viscoelasticity of material. In this case, are possible two in principle different approaches.

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The first - phenomenological - consists in the laboratory investigation of ready material for the target/purpose of the determination of its mechanical characteristics and use of known equations of anisotropic shell.

Second approach is connected with the macrostructural analysis of material. It makes it possible to express the mechanical characteristics of material as a whole by the appropriate characteristics of its components: bonding agent and fittings, which makes it possible to predict the properties of composite material depending on its structural parameters (solidity/loading factor, the orientation of fittings, and so forth), and consequently, to solve the problems of optimum planning.

The complete system of equations of thin-walled shell consists of three groups of the equations: 1) the equation of equilibrium or motion; 2) the geometric equations of the shells which are derive/concluded on the basis of the determined assumptions about the character of strain and significantly are connected with the geometric parameters of shell (to choose from of these assumptions they affect the property of the material of shell); 3) the physical equations, which relate voltage/stresses with strains and the reflecting properties of material. In this group of equations, finds their reflection of the special feature/peculiarity of the mechanics

of polymeric material.

The special feature/peculiarity reinforced plastic is the anisotropy of their deformation properties with low shift rigidity both in the plane of layers of the reinforcing fabric and between layers of fabric. Because of this the standard of cell/element in the process of bending does not remain rectilinear, but bent. For real constructions made of the oriented glass-fiber-reinforced plastics of the ratio of Young's modulus to shear modulus, they can be such, that even for engineering calculations the failure of the hypothesis of undeformable standards becomes necessary.

Direct measurement of the bending of the cross sections of rods from the oriented glass-fiber-reinforced plastics with elongation and bending is carried out in works [175], where on the basis of experiments is shown the need for the calculation of rods from these materials on the refined formulas, without the use of the simplifying hypothesis of flat/plane section/cuts.

The first, most complete experiments on the stability of glass-plastic shells with different external loads are carried out by A. A. Bushtyrkov [176-178]. In these investigations the glass-fiber-reinforced plastic is considered as elastic-orthotropic material.

The carried out experiments confirm the possibility of using the theory of orthotropic plates and shells for practical calculations for stability. Unlike isotropic shells, the stability factor of orthotropic shells is not constant, but it depends substantially on the relationship/ratio of the elastic constants of material.

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To each form of anisotropy corresponds its value both of upper and lower critical load. By the investigations of A. A. Bushtyrkova is clearly shown the effect of the anisotropy of material together with the geometric parameters of shell on the critical value of load and wave formation.

In A. A. Bushtyrkova's curent works [172] is used the property of shells of glass-fiber-reinforced plastic to accept small residual strains and to retain them for some time. By the repeated loading of one shell of glass-fiber-reinforced plastic it was possible to obtain the values of critical loads at several values of the depth of initial dent. The results of the carried out experiments attest to the fact that the theory, constructed on the similarity of initial and additional of sagging/deflections, qualitatively describes the

results of experiments and that the loss of stability occurs at the value of upper critical load taking into account initial imperfections.

A. A. Bushtyrkov investigated also the supercritical stress-strain state of shells for the target/purpose of the determination of the breaking load for thin-walled construction and its supercritical rigidity. This question is especially actual because fiber glass plastic with short-term loading possess the almost modern elasticity up to the torque/moment of destruction. By the mentioned author it is shown, that with the aid of nonlinear theory is possible in principle research on the stress-strain state of cylindrical fiber glass plastic shells in supercritical stage. This is very important in connection with the fact that in supercritical state the glass-plastic shells can bear the considerable loads, close to upper critical or more them.

Further by A. A. Bushtyrkov studied supercritical stress-strain state of square orthotropic plate from glass-fiber-reinforced plastic is shown, that within known limits experimental amounts of deflection will agree well with theoretical and that with the aid of nonlinear theory even in initial approach/approximations completely satisfactorily is predicted the character of a change in the stresses in plate [180].

In the works of V. S. Gumenyuk, V. S. Kravchuk and V. V. Lushik [181] also is utilized the property of glass-fiber-reinforced plastic to retain elastic properties during comparatively large strains. It is indicated that to study the stability of shell during the emergence of large radial displacements is possible only in such a case, when the material of shell possesses high elastic limit, since otherwise appear plastic deformations.

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For the experimental investigation of the behavior of the cylindrical shell, compressed along axle/axis, in supercritical state was tested shell of the glass-fiber-reinforced plastic PSM-103 (structure one-to-one), which retains elastic properties during deformations 1-3o/o. The diameter of shell 300 mm, length 600 mm, thickness 1 mm, end/faces are intensified by the longitudinal and circular layers with a length of 40 mm, by a thickness of 4 mm. Bending characteristics of material $E_1 = E_2 = 2.1 \cdot 10^5$ kg/cm², $\mu_1 = \mu_2 = 0.2$. Was used the testing machine, making it possible to regulate and to check the speed of loading and to record/write diagram load - the displacement/movement of end/faces. In the process of loading, the shell lost stability with sharp knock. The maximum value of the

applied force was considered upper critical force, lower critical force was determined with the unloading of shell. Diagram load, the displacement/movement of end/faces, is qualitatively similar of the theoretical, obtained for the shell ideal form in geometrically nonlinear setting, moreover the experimental values of upper and lower critical forces are repeated with the considerable number of repeated loadings.

The further theoretical and experimental analyses of stability of glass-plastic shells are carried out by V. V. Ivanov, by L. N. Smetanina, etc. [182, 183] under A. S. Vol'mir's management.

By V. K. Ivanov it is shown, that the bulge of orthotropic cylindrical glass-plastic shells during axial compression represents by itself dynamic process. Transfer/transition from one state of equilibrium to another is realize/accomplished by means of knocks. Shells lose stability, forming the axially nonsymmetric diamond-shaped bulge whose character sharply is changed depending on the relation of the moduli of elasticity along generatrix and arc. If module/modulus along arc is more, then also bulges are elongated along arc, and vice versa. The effect of nonlinearity of orthotropic glass-plastic shells render/showed smaller than in isotropic metallic. On the value of critical load, essential effect exerts the parameter, i.e., the square root from the main normal moduli of

elasticity, divide by shear modulus, i.e., the parameter, which characterizes the properties of material. With decrease in this parameter, the critical load grows. Consequently, for an increase in the critical load it is necessary to increase shear modulus, which is reached by applying resins with the increased cohesive strength and the increased adhesion to fiberglass. Real critical load is located, as a rule, between upper and lower theoretical loads. For practical calculations is recommended the approximaticn formula.

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Experimental and theoretical studies of the stability of flexible orthotropic plate with free edges is carried out by L. G. Butakovoy [184]. Were tested glass-plastic plates 200 x 200 x 4 mm in size/dimension of, also, with the relation of layers 1:4. Is establish/installed critical load and curved of supercritical deformation under the effect of distributed and concentrated loads. Theoretical calculation is carried out by the method successive carrying out of with the use of finite differences in electronic digital computer BESM-2M. During the comparison of the results of the solution with experimental data obtained satisfactory agreement.

All the examined works are carried out with the use of a hypothesis of direct/straight standards. In V. I. Korolev's monograph

[186] is proposed the approximation method of the account of the effect of the interlayer shift/shears of laminar anisotropic plates and shells. On the base of this method, is investigated the stability of anisotropic rectangular plates. Are examined in detail some characteristic tasks of the stability of orthotropic sandwich plates and shells with elastic fillers, and also are given tasks by choice of the optimum structure of laminated plastic of cylindrical shell.

V. T. Tomashevskiy [187, 187a] also is focused attention on the need for the account of the anisotropy of glass-fiber-reinforced plastic on the third coordinate, i.e., it indicates that the limits of the applicability of the hypothesis of direct/straight standards in the case of anisotropic materials depend not only on the geometry of construction, but also on the relationship/ratio of elastic constants. Of glass-fiber-reinforced plastics with interlamination shift/shear, the reinforcing filaments relatively weakly are involved into work and the effort/forces of transverse shift/shear in larger measure are absorbed by bending agent.

To evaluate the effect of such most characteristic for a glass-fiber-reinforced plastic special feature/peculiarities as elastic anisotropy and compliance/pliability with respect to interlayer shift/shear, was investigated the stressed state of circular cylindrical shell of glass-fiber-reinforced plastic,

reinforced equidistant heel rings, under the action of axisymmetric and column load. Is obtained the system of equations of the equilibrium of the shell whose solution is given under the boundary conditions, which consider the consistency of the deformation of shell and fin/edges.

By means of the analysis of the results of the solution determined the limits of the applicability of the theory of films for such type of constructions, is given evaluation of the effect of the anisotropy of material. Since the material is created simultaneously with the development of construction, are given the recommendations by choice of the optimum schematics of the reinforcement of the shell of cylinder, which make it possible to facilitate construction or to increase its bearing capacity.

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In recent years attention drew the works, relating to the dynamic stability of glass-plastic plates and shells, which are located under the action of intermittent load [188, 189], in the flow of gas or liquid [190] with the rapidly accompanying and impact loads of aperiodic character. The works of the last/latter direction, carried out under A. S. Vol'mir's management/manual [191], can be divided into two groups.

1. With strictly dynamic load, when process of bulge is contiaued during period, considerably larger propagation time of elastic wave along construction, they disregard wavelike nature of transmission of elastic deformations. In this setting are carried out the investigations of L. N. Smetaninoy [191].

2. Second group of works is related to tasks with impact load, in which period of propagation of elastic waves along construction let us compare in the course of time prior to beginning of violent bulge; in these tasks it is necessary to consider wavelike nature of transmission of elastic deformations. The stability of glass-plastic shells with impact load is investigated by V. L. Agamirov and L. N. Smetaminoy [193]. This and other works showed that with an increase in the velocity of loading are developed the higher forms of loss of stability and as consequence grow/rises the value of critical load.

Let us further examine the investigations, considering the rheonomic properties of polymeric materials.

The problems of the prolonged stability of plates and shells of polymeric materials are discussed in the review of S. N. Rabotnova at I Riga conference on the mechanics of polymers in 1965.

The main directions of development, posing of the question and the results of the solution of the problems of stability during creep are given in the works of A. R. Rzhanitsin [195, 196], of S. N. Rabotnova [127, 156], of S. A. Shesterikova [129, 199, 200], of A. S. Vol'mir [65], of L. M. Kurshin [201, 202], of G. V. Ivanov [97, 203] et al. They all, however, are related mainly to the analysis of stability of cell/elements from traditional materials (metal, concrete, etc.). There is thus far of a little works on the prolonged stability structural cell/elements from polymeric materials. In this connection it is possible to note S. N. Rabotnova's report [194], G. I. Brizgalin's works [204, 205], E. N. Sinitzin [213], A. G. Teregulova [214], P. M. Ogibalova [207], M. A. Koltunova [207, 208], A. Ali-El'-Kurmani [208] et al.

The study of the prolonged stability of fine/thin rectangular plate from glass-fiber-reinforced plastic taking into account shift creep in the plane of plate is carried out by G. I. Brizgalin.

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By it is examined the supported from four sides plate to which along two opposite sides is applied the evenly distributed compressive load

with an intensity of P. Creep in the direction of reinforcement they disregard. Since glass-fiber-reinforced plastic is the material whose anisotropy can be within certain limits changed arbitrarily, is placed the question concerning the most rational reinforcement of plate. If we in the fixed/recorded sense of sides a/b change the moduli of elasticity along principal directions E_1 and E_2, then critical load will be substantially change. it is proposed the method of the determination of this relationship/ratio between module/moduli and a quantity of reinforcement with which prolonged critical load takes greatest value. It is shown, that the optimum reinforcement can increase critical load several times.

By M. A. Koltunov and A. El'-Kurmani is examined the stability of the compressed in axial direction closed circular cylindrical shell, manufactured from orthotropic glass-fiber-reinforced plastic from that which reinforces the fabric of linen weave. The weft of fabric is directed along generatrix, and basis - on guide. Is taken into account the linear heredity of material in the form of Boltzmann-Volterra. By authors were obtained elastic upper and lower critical forces, and also similar expressions taking into account heredity. It is further noted that the account of linear heredity decreases the value of critical load for shells of glass-fiber-reinforced plastic. Is given the procedure of the determination of critical time and it is shown, that the critical

loads of the shells whose material possesses linear heredity, depend substantially on the mode/conditions of loading, after grow/rising with an increase in the velocity of loading.

Such a conclusion is confirmed by the experimental investigations of shells of some types of polymeric materials.

P. M. Ogibalov and M. A. Koltunov [207] studied the stability of plates and shells taking into account of rheonomicity of the mechanical properties of glass-fiber-reinforced plastics established that the critical sagging/deflections with constant load increase. The values of "upper" and "lower" critical loads for viscoelastic shells depend substantially on the velocity of the loading: with an increase in the velocity, the value of upper critical load is raised.

The problems of cracking during creep of slanting spherical shells of polymeric materials are solved by Ye. Tungl and Khuan Nay-Chen in connection with shell of polymethyl methacrylate [209, 298]. In these works determined critical time.

The stability of thin-walled cell/elements from the reinforced polymeric materials taking into account their macrostructure is investigated by V. V. Bolotin and his pupils on the base of the proposed to them theory of reinforced media [209, 210].

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The fundamental equations of the theory of the reinforced media are derive/concluded from the macrostructural considerations, obtained by means of the reinforcement of certain viscoelastic medium by the large number of linear or surface elastic cell/elements of high strength and rigidity. The stress-strain state of each of the reinforcing cell/elements is considered taking into account interaction with connecting material. Further by means of the operation of smearing inhomogeneous medium is approximated by certain energetically equivalent quasi-homogeneous medium. This approach is generalized also for the case when medium is reinforced by the slightly bent elastic layers how it is possible to explain the known experimental fact, which consists in a reduction/descent in the moduli of elasticity of laminar glass-fiber-reinforced plastics.

On the basis of the theory of the reinforced media, V. V. Bolotin and V. N. Moskalenko [212] derived the fundamental equations of the theory of plates and shells, made from the reinforced materials. Is establish/installed also the need for failure of Kirchhoff-Love's hypothesis during the construction of the theory of plates and shells, made from the medium in question. The authors show

communication/connection of the obtained by them relationship/ratios with the theory of anisotropic plates and shells, and also with the theory of plates and shells of the medium of Voigt-Kosser give solutions of the stability of shells.

In Ye. N. Sinitzin's works [213] on the basis of the theory of the reinforced media is investigated the bulge of the flattened rods and plates from the viscoelastic reinforced material under the action of longitudinal forces. By variational method are derive/concluded the corresponding differential-difference equations and boundary conditions (2n equations for rods, 3n equations for plates, moreover n is a number of reinforcing layers). With the aid of the "principle of the smearing of energy" of the system of differential-difference equations, they are replaced by the equivalent system of partial differential equations (2n equations for rods, 3n equations for plates). Are given the precise and approximation methods of the solution of the problems of the bulge of the viscoelastic reinforced rods and plates. D. N. Sinitzin it explains the condition of the applicability of the refined theories of the bending of uniform rods and plates and it shows that the effect of the deformations of transverse shift/shear is more fully considered by the theory of multilayer rods. In the work is examined the process of the bulge of rod, which is accompanied by relaxation of initial stresses, and is emphasized the significant role of viscoelastic shearing strains.

Besides strict methods of the theory of the reinforced media, property of the anisotropy of the viscoelastic behavior of glass-fiber-reinforced plastics, and in particular its low transverse shift rigidity, they can be taken into account by the introduction of the corresponding kinematic hypotheses. This approach is based on the construction of the two-dimensional theories of plates and shells during the use of kinematic hypotheses, less rigid, than Kirchhoff-Love's classical hypothesis. By this method it is possible to obtain not the too complex equations, which yield to treatment and resolution in concrete/specific/actual engineer missions. The successful hypotheses, which consider the deformations of transverse shift/shear, were proposed to S. P. Timoshenko, Ye. Reyssner, B. F. Blasov, S. A. Ambartsumyan, M. P. Sheremet'yev, B. L. Pelekh, and other researchers.

In A. G. Teregulova's work [214] is analyzed the effect of the deformations of transverse shift/shear on the stress-strain state of plates from the oriented glass-fiber-reinforced plastics in geometric nonlinear setting with bending and in the tasks of stability. The deformations of transverse shift/shear are considered according to the procedure, proposed by S. A. Ambartsumyan and D. V. Peshtmaldzhan [215]. The viscoelastic properties of material with shift/shear are

described by the linear theory of heredity, property with elongation - compression along principal directions they are considered elastic. Operational expressions for a shift/shear, according to the works of A. L. Rabinovich and P. N. Verkhovskogo, are expressed approximately through the appropriate operators of bonding agent. By the method of Navier and by using the integral operator of S. N. Rabotnova is solved the problem of the bending of rectangular plate under the action of the evenly distributed load. Assuming that the process of bulge is the development of low initial inaccuracies, A. G. Teregulov examined the stability of the rectangular plate, compressed in two directions. After accepting for critical state the torque/moment of reduction to zero accelerations of sagging/deflection, it obtained equation for determining critical time.

The rheonomic properties of the broad class of polymeric materials (glass-fiber-reinforced plastics, Textolite, etc.) for the case of the moderate stresses are described by the linear relationship/ratios of orthotropic and uniform material. When the shift rigidity of plate or shells is sufficient, in the tasks of stability during creep of plates and shells of orthotropic and isotropic materials to admissibly use Kirchhoff-Love's hypothesis; however in many encountered in practice cases the shift rigidity of materials is insufficient, in consequence of which standard in the process of deformation is bent, which is led to the need for the

account of transverse shift/shears during the analysis of stability of plates and shells. Deformation properties of some polymeric materials at all stress levels nonlinear.

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When us interests only the stability of system during its low deviations from basic condition/position, we utilize the method of the variation of the equation of deformation properties, used for the first time by Yu. N. Rabotnov and S. A. Shesterikov [127].

In present chapter we investigate the stability of plates and shells of orthotropic material during linear creep on the base of the refined kinematic model of the type of Timoshenko and will establish/install the effect of transverse shift/shears on critical loads. Let us examine the stability of plates and shells of nonlinear-creeping, initial-isotropic material in cases when reverse/inverse creep is described by the law of creep with loading and when reverse/inverse creep is described by the law, which differs from preceding/previous. This investigation is carried out by the integration of the different forms of the function of local deformations for the appropriate regions of sphere.

Some experiments to the prolonged stability of cylindrical

shells of polyethylene establish/installed the applicability of the obtained theoretical relationship/ratios.

§5.2. Initial physical relationship/ratios, utilized for the solution of particular problems.

The deformation properties of the broad class of polymeric materials can be approximated by the relationship/ratios of uniform, orthotropic, linear viscoelastic material.

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In connection with the tasks of the stability of plates and shells the stresses of σ_{33} we disregard; then we are have following relationship/ratios (see formulas in chapter I, Section 1.2.4):

$$\begin{aligned}
\varepsilon_{11} &= a_{1111}\sigma_{11} + a_{1122}\sigma_{22} + \int_0^t K_{1111}(t-\theta)\sigma_{11}(\theta)\,d\theta + \\
&+ \int_0^t K_{1122}(t-\theta)\sigma_{22}(\theta)\,d\theta; \\
\varepsilon_{22} &= a_{2211}\sigma_{11} + a_{2222}\sigma_{22} + \int_0^t K_{2211}(t-\theta)\sigma_{11}(\theta)\,d\theta + \\
&+ \int_0^t K_{2222}(t-\theta)\sigma_{22}(\theta)\,d\theta; \\
\varepsilon_{12} &= a_{1212}\sigma_{12} + \int_0^t K_{1212}(t-\theta)\sigma_{12}(\theta)\,d\theta;
\end{aligned} \tag{5.2.1}$$

$$\varepsilon_{13}=a_{1313}\sigma_{13}+\int_0^t K_{1313}(t-\theta)\sigma_{13}(\theta)\,d\theta;$$

$$\varepsilon_{23}=a_{2323}\sigma_{23}+\int_0^t K_{2323}(t-\theta)\sigma_{23}(\theta)\,d\theta,$$

or, after solving relatively σ_{ij}:

$$\begin{aligned}
\sigma_{11}&=A_{1111}\varepsilon_{11}+A_{1122}\varepsilon_{22}-\int_0^t R_{1111}(t-\theta)\varepsilon_{11}(\theta)\,d\theta-\\
&\quad-\int_0^t R_{1122}(t-\theta)\varepsilon_{22}(\theta)\,d\theta;\\
\sigma_{22}&=A_{2222}\varepsilon_{22}+A_{2211}\varepsilon_{11}-\int_0^t R_{2222}(t-\theta)\varepsilon_{22}(\theta)\,d\theta-\\
&\quad-\int_0^t R_{2211}(t-\theta)\varepsilon_{11}(\theta)\,d\theta;\\
\sigma_{12}&=A_{1212}\varepsilon_{12}-\int_0^t R_{1212}(t-\theta)\varepsilon_{12}(\theta)\,d\theta;\\
\sigma_{13}&=A_{1313}\varepsilon_{13}-\int_0^t R_{1313}(t-\theta)\varepsilon_{13}(\theta)\,d\theta;\\
\sigma_{23}&=A_{2323}\varepsilon_{23}-\int_0^t R_{2323}(t-\theta)\varepsilon_{23}(\theta)\,d\theta.
\end{aligned}\tag{5.2.2}$$

In many in practice actual/urgent cases with creep and direction of reinforcement it is possible without essential error to disregard [154, 205].

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If one assumes that the direction of reinforcement coincides with the direction of deformations ε_{11} and ε_{22} from (5.2.1), for this case there is:

$$\varepsilon_{11}=a_{1111}\sigma_{11}+a_{1122}\sigma_{22};$$

$$\varepsilon_{22}=a_{2222}\sigma_{22}+a_{2211}\sigma_{11};$$

$$\begin{aligned}\varepsilon_{12}&=a_{1212}\sigma_{12}+\int_0^t K_{1212}(t-\theta)\,\sigma_{12}(\theta)\,d\theta;\\ \varepsilon_{13}&=a_{1313}\sigma_{13}+\int_0^t K_{1313}(t-\theta)\,\sigma_{13}(\theta)\,d\theta;\\ \varepsilon_{23}&=a_{2323}\sigma_{23}+\int_0^t K_{2323}(t-\theta)\,\sigma_{23}(\theta)\,d\theta,\end{aligned} \tag{5.2.3}$$

or from (5.2.2):

$$\begin{aligned}
&\sigma_{11}=A_{1111}\varepsilon_{11}+A_{1122}\varepsilon_{22};\\
&\sigma_{22}=A_{2222}\varepsilon_{22}+A_{2211}\varepsilon_{11};\\
&\sigma_{12}=A_{1212}\varepsilon_{12}-\int_0^t R_{1212}(t-\theta)\,\varepsilon_{12}(\theta)\,d\theta;\\
&\sigma_{13}=A_{1313}\varepsilon_{13}-\int_0^t R_{1313}(t-\theta)\,\varepsilon_{13}(\theta)\,d\theta;\\
&\sigma_{23}=A_{2323}\varepsilon_{23}-\int_0^t R_{2323}(t-\theta)\,\varepsilon_{23}(\theta)\,d\theta.
\end{aligned} \tag{5.2.4}$$

For the case when the deformation properties of polymer can be described by the relationship/ratios of isotropic viscoelastic material, we have following expressions (for the flat/plane case):

$$\begin{aligned}
&\varepsilon_{11}=\frac{1}{H}(\sigma_{11}-\mu\sigma_{22})+\int_0^t K(t-\theta)(\sigma_{11}-\mu\sigma_{22})\,d\theta;\\
&\varepsilon_{22}=\frac{1}{H}(\sigma_{22}-\mu\sigma_{11})+\int_0^t K(t-\theta)(\sigma_{22}-\mu\sigma_{11})\,d\theta;\\
&\varepsilon_{12}=\frac{2(1+\mu)}{H}\sigma_{12}+2(1+\mu)\int_0^t K(t-\theta)\,\sigma_{12}\,d\theta,
\end{aligned} \tag{5.2.5}$$

or relatively σ_{ij}:

$$\sigma_{11}=\frac{1}{1-\mu^2}H(\varepsilon_{11}+\mu\varepsilon_{22})-\frac{1}{1-\mu^2}\int_0^t R(t-\theta)(\varepsilon_{11}+\mu\varepsilon_{22})\,d\theta;$$

$$\sigma_{22}=\frac{1}{1-\mu^2}H(\varepsilon_{22}+\mu\varepsilon_{11})-\frac{1}{1-\mu^2}\int_0^t R(t-\theta)(\varepsilon_{22}+\mu\varepsilon_{11})d\theta;$$

$$\sigma_{12}=H=\frac{\varepsilon_{12}}{2(1+\mu)}-\frac{1}{2(1+\mu)}\int_0^t R(t-\theta)\varepsilon_{12}(\theta)d\theta. \quad (5.2.6)$$

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The number of polymeric materials possesses nonlinear deformation properties in time, moreover in many instances it has different laws with loading and unloading. If we are interested in the stability of system only during low deviations from its ground state, then it is possible to use the method of a variation in the equation of state.

In the case when the deformation properties of material are identical with loading and unloading, for an increase in the deformations, we have

$$\delta e_{ij}=\frac{1}{S}\iint_S(\delta\gamma_{xz}v_{ij}+\gamma_{xz}\delta v_{ij})ds, \quad (5.2.7)$$

where integration it is spread to entire surface of the sphere: $\delta\gamma_{xz}$

is increase of the function of local deformations; δv_{ij} is increase of weighting function.

For a material with different properties with loading and unloading, we have for increases the following initial dependence:

$$\delta e_{ij}=\frac{1}{S}\iint_{S^{+}}(\delta\gamma_{xz}v_{ij}+\gamma_{xz}\delta v_{ij})\,ds+\frac{1}{S}\iint_{S^{-}}(\delta\bar{\gamma}_{xz}v_{ij}+\delta v_{ij}\bar{\gamma}_{xz})\,ds. \tag{5.2.8}$$

Here the first integral is spread to the region of sphere $S^{(+)}$, where it is fulfilled the condition of additional charge, by the second - to region S^{-} with unloading.

§5.3. Stability of isotropic plate.

Let us examine the rectangular plate with initial sagging/deflection, manufactured from isotropic viscoelastic polymeric material, which is subordinated to relationship/ratios (5.2.6). Plate is compressed in two directions and on it affect tangential forces along outline/contour.

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Let us introduce the common assumptions of the technical theory

of the bending of the plates:

a) the totality of the points, which lie/rested up to the deformation of plate on any straight line, perpendicular to median surface, remains on straight line, normal to this surface and after the deformation of plate;

b) are disregarded normal stresses of σ_{33} on the pads, parallel to medium plane.

The coordinate plane XOY let us count the coinciding with medium plane plate, and z axis it is directed down.

Communication/connection between deformations and sagging/deflections with the adopted assumptions of the technical theory of the bending of plates takes the form:

$$\begin{aligned} e_{11} &= -z\frac{\partial^2 w}{\partial x^2};\\ e_{22} &= -z\frac{\partial^2 w}{\partial y^2};\\ e_{12} &= -2z\frac{\partial^2 w}{\partial x \partial y}, \end{aligned} \qquad (5.3.1)$$

where w - the sagging/deflection of plate; z - the distance of point from median surface.

If we sum up elementary torque/moments from an entire height/altitude of plate, we will obtain communication/connection between torque/moments and components of the deformations of median surface:

$$M_x = -BH\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) + B\int_0^t R(t-\theta)\left(\frac{\partial^2 w}{\partial x^2} - \mu\frac{\partial^2 w}{\partial y^2}\right)d\theta;$$

$$M_y = -BH\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) + B\int_0^t R(t-\theta)\left(\frac{\partial^2 w}{\partial y^2} - \mu\frac{\partial^2 w}{\partial x^2}\right)d\theta;$$

$$M_{xy} = -(1-\mu)BH\frac{\partial^2 w}{\partial x\partial y} + (1-\mu)B\int_0^t R(t-\theta)\frac{\partial^2 w}{\partial x\partial y}d\theta; \quad (5.3.2)$$

$$B = \frac{h^2}{12(1-\mu^2)}$$

The equations of the equilibrium of the plate on which affect tangential forces and compressive forces in two directions and which has initial curvature, acquire the following known form:

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} = -\left(N_x\frac{\partial^2(w_0+w)}{\partial x^2} + N_y\frac{\partial^2(w_0+w)}{\partial y^2} - 2N_{xy}\frac{\partial^2(w_0+w)}{\partial x\partial y}\right), \quad (5.3.3)$$

where w_0 - the initial bending of plate; N_x, N_y and N_{xy} -

compressive and the tangent of effort/force.

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After substituting in the equation of equilibrium (5.3.3) values obtained above of torque/moments, according to (5.3.2), after conversions we will obtain the equation of the flexure of the plate

$$BH\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-B\left[\int_0^t R(t-\theta)\frac{\partial^4 w}{\partial x^4}d\theta+\right.$$

$$\left.+2\int_0^t R(t-\theta)\frac{\partial^4 w}{\partial x^2\partial y^2}d\theta+\int_0^t R(t-\theta)\frac{\partial^4 w}{\partial y^4}d\theta\right]= \quad (5.3.4)$$

$$=-\left(N_x\frac{\partial^2(w_0+w)}{\partial x^2}+N_y\frac{\partial^2(w_0+w)}{\partial y^2}-2N_{xy}\frac{\partial^2(w_0+w)}{\partial x\partial y}\right).$$

Let us assume that the initial sagging/deflection of plate is determined by the equation

$$w_0=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \quad (5.3.5)$$

where a and b are lengths of the sides of plate.

The solution to the equation of sagging/deflections with constant external loads let us search for in the following form:

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}. \quad (5.3.6)$$

Now the equation of the sagging/deflections of signs the form

$$BH\left(\frac{\pi}{a}\right)^4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m^4A_{mn}(t)+2BH\frac{\pi^4}{a^2b^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m^2n^2A_{mn}(t)+$$
$$+BH\left(\frac{\pi}{b}\right)^4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}n^4A_{mn}(t)-B\pi^4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2\times$$
$$\times\int_0^t R(t-\theta)A_{mn}(\theta)d\theta=N_x\left(\frac{\pi}{a}\right)^2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}m^2[A_{mn}(t)+a_{mn}]+$$
$$+N_y\left(\frac{\pi}{b}\right)^2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}n^2[A_{mn}(t)+a_{mn}]+$$
$$+2N_{xy}\frac{\pi^2}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}mn\left[a_{mn}+A_{mn}(t)\right]\operatorname{ctg}\frac{m\pi x}{a}\operatorname{ctg}\frac{n\pi y}{b}. \quad (5.3.7)$$

For determining any coefficient $A_{mn}(t)$ series (5.3.6) let us accept nucleus in the form of exponential dependence, sometimes utilized for description creep of the polymeric materials:

$$K(t-\theta)=\frac{H-E}{H^2\alpha}e^{-\frac{E(t-\theta)}{H\alpha}}. \quad (5.3.8)$$

The resolvent of the taken nucleus it will be

$$R(t-\theta)=\frac{H-E}{\alpha}e^{-\frac{t-\theta}{\alpha}}. \quad (5.3.9)$$

Under these assumptions for determining coefficient $A_{mn}(t)$ we obtain the following differential equation:

$$\alpha(N_{mn}{}^{\text{M}}-N_{mn})\dot{A}_{mn}(t)+(N_{mn}{}^{\text{Д}}-N_{mn})A_{mn}(t)-N_{mn}a_{mn}=0,\quad (5.3.10)$$

where

$$N_{mn}=N_x\left(\frac{m}{a}\right)^2+N_y\left(\frac{n}{b}\right)^2+2N_{xy}\frac{mn}{ab}\operatorname{ctg}\frac{m\pi x}{a}\operatorname{ctg}\frac{n\pi y}{b};$$

$$N_{mn}{}^{\text{M}}=BH\pi^2\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2;$$

$$N_{mn}{}^{\text{Д}}=BE\pi^2\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2.$$

By solving of equation (5.3.10) under the initial condition

$$A_{mn}(t=0)=\frac{N_{mn}a_{mn}}{N_{mn}{}^{\text{M}}-N_{mn}}\qquad (5.3.11)$$

we obtain dependence for the arbitrary coefficient of a series (5.3.6):

$$A_{mn}(t)=\frac{N_{mn}a_{mn}}{N_{mn}{}^{\text{Д}}-N_{mn}}+$$

$$+\left(\frac{N_{mn}a_{mn}}{N_{mn}{}^{\text{M}}-N_{mn}}-\frac{N_{mn}a_{mn}}{N_{mn}{}^{\text{Д}}-N_{mn}}\right)e^{-\frac{N_{mn}{}^{\text{Д}}-N_{mn}}{N_{mn}{}^{\text{M}}-N_{mn}}\frac{t}{\alpha}}.\qquad (5.3.12)$$

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During satisfaction of condition $N_{mn}<N_{mn}^{Д}$ the rate of the increase of sagging/deflections in time attenuates, when $N_{mn}^{Д}<N_{mn}<N_{mn}^{M}$ the velocity of sagging/deflection grow/rises, when $N_{mn}=N_{mn}^{M}$ the sagging/deflection goes to infinity with t = 0, when $N_{mn}=N_{mn}^{Д}$ the sagging/deflection grow/rises at constant velocity.

§5.4. Stability is circular cylindrical shell of orthotropic material during creep with the use of a kinematic model of Kirchhoff - Love.

We will consider a round cylindrical shell, compressed along by the forming constant force N_{10}. The properties of material (in planar-stressed state) let us accept in more general view, than in the preceding/previous paragraph: let us assume that the shell is made from orthotropic material according to dependences (5.2.2).

At zero time, let us give to shell disturbance/perturbation in the form of low initial sagging/deflection of the axisymmetric form:

$$w=f\sin\frac{m\pi x}{l} \qquad (5.4.1)$$

it is investigated the further behavior of sagging/deflection. The state of shell will be stable, if initial sagging/deflection decreases, and unstable, if initial sagging/deflection in time

unlimitedly increases. Since motion during the creep of the material here sufficiently slow, inertial forces we disregard. In the disturbed state for a shell besides physical relationship/ratios (5.2.2) we have equations of equilibrium and consistency of deformations (with Kirchhoff-Love's taken hypothesis):

$$\frac{\partial^2 M_1}{\partial x^2}+2\frac{\partial^2 H}{\partial x\partial y}+\frac{\partial^2 M_2}{\partial y^2}-N_{10}\frac{\partial^2 w}{\partial x^2}+\frac{N_2}{R}=0; \qquad (5.4.2)$$

$$\frac{\partial^2 \varepsilon_1}{\partial y^2}+\frac{\partial^2 \varepsilon_2}{\partial x^2}-\frac{\partial^2 \gamma}{\partial x\partial y}+\frac{1}{R}\frac{\partial^2 w}{\partial x^2}=0. \qquad (5.4.3)$$

Let us introduce the function of the stresses

$$N_1=\sigma_x h=\frac{\partial^2\varphi}{\partial y^2},\quad N_2=\sigma_y h=\frac{\partial^2\varphi}{\partial x^2},\quad S=\tau_{xy}h=-\frac{\partial^2\varphi}{\partial x\partial y}. \qquad (5.4.4)$$

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Since the form of the bulge of shell is axisymmetric, effort/forces in (5.4.2) depend only on coordinate x. For M_1 we have the expression

$$M_1=\int_{-0.5h}^{0.5h}\sigma_x z\,dz=-A_{11}\frac{h^3}{12}\frac{\partial^2 w}{\partial x^2}-A_{12}\frac{h^3}{12}\frac{\partial^2 w}{\partial y^2}+\frac{h^3}{12}\int_0^t R_{11}(t-\theta)\times$$

$$\times\frac{\partial^2 w}{\partial x^2}d\theta+\frac{h^3}{12}\int_0^t R_{12}(t-\theta)\frac{\partial^2 w}{\partial y^2}d\theta. \qquad (5.4.5)$$

It is here taken into account, that $\varepsilon_{11}=-z\frac{\partial^2 w}{\partial x^2}$, $\varepsilon_{22}=-z\frac{\partial^2 w}{\partial y^2}$, $\varepsilon_{12}=-2z\frac{\partial^2 w}{\partial x\partial y}$. are accepted the matrix notations of indices.

From (5.4.2) we obtain

$$\frac{1}{R}\frac{\partial^2\varphi}{\partial x^2}=A_{11}\frac{\partial^4 w}{\partial x^4}\frac{h^3}{12}-\frac{h^3}{12}\int_0^t R_{11}(t-\theta)\frac{\partial^4 w}{\partial x^4}d\theta+N_{10}\frac{\partial^2 w}{\partial x^2}. \quad (5.4.6)$$

Accepting $R_{11}(t-\theta)$ in the form of exponential $R_{11}(t-\theta) = 1/n_{11} \times A^*_{11}e^{-\frac{t-\theta}{n_{11}}}$, I pass from integral equation (5.4.6) to differential:

$$\frac{1}{R}\dot{\varphi}_{xxxx}+\frac{1}{n_{11}R}\varphi_{xxxx}-A_{11}\dot{w}_{xxxxxx}\frac{h^3}{12}-N_{10}\dot{w}_{xxxx}-$$
$$-\frac{h^3}{12n_{11}}(A_{11}-A^*_{11})w_{xxxxxx}-\frac{1}{n_{11}}N_{10}w_{xxxx}=0. \quad (5.4.7)$$

Utilizing further in the equation of the consistency of deformations (5.4.3) physical relationship/ratios (5.2.1) and introducing the function of stresses, we have

$$\frac{a_{22}}{h}\frac{\partial^4\varphi}{\partial x^4}+\frac{1}{h}\int_0^t K_{22}(t-\theta)\frac{\partial^4\varphi}{\partial x^4}d\theta+\frac{1}{R}\frac{\partial^2 w}{\partial x^2}=0. \quad (5.4.8)$$

Is here considered, that with axisymmetric form all variables depend only on coordinate x.

From this equation with $K_{22}(t-\theta) = \frac{1}{\alpha_{22}}a^*e^{-\frac{t-\theta}{\alpha_{22}}}$ and pass to differential:

$$\frac{\partial^4\dot{\varphi}}{\partial x^4}+\frac{a^*_{22}+a_{22}}{\alpha_{22}a_{22}}\frac{\partial^4\varphi}{\partial x^4}+\frac{h}{Ra_{22}}\frac{\partial^2\dot{w}}{\partial x^2}+\frac{h}{\alpha_{22}a_{22}R}\frac{\partial^2 w}{\partial x^2}=0. \quad (5.4.9)$$

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Eliminating from equations (5.4.7) and (5.4.9) ϕ and taking into account that

$$\frac{\partial^6 w}{\partial x^6}=-f\left(\frac{m\pi}{l}\right)^6\sin\frac{m\pi x}{l};$$
$$\frac{\partial^4 w}{\partial x^4}=f\left(\frac{m\pi}{l}\right)^4\sin\frac{m\pi x}{l};\qquad(5.4.10)$$
$$\frac{\partial^2 w}{\partial x^2}=-f\left(\frac{m\pi}{l}\right)^2\sin\frac{m\pi x}{l},$$

for f we obtain the equation

$$a_2\ddot{f}+a_1\dot{f}+a_0 f=0,\qquad(5.4.11)$$

where

$$a_2=\frac{\alpha_{22}a_{22}n_{11}}{n_{11}(a^*_{22}+a_{22})-\alpha_{22}a_{22}}\left[RA_{11}\frac{h^3}{12}\left(\frac{\pi}{l}\right)^6+\right.$$
$$\left.+\frac{h}{Ra_{22}}\left(\frac{\pi}{l}\right)^2-RN_{10}\left(\frac{\pi}{l}\right)^4\right];$$
$$a_1=D+\frac{a_2}{n_{11}}+\frac{h^3}{12}RA_{11}\left(\frac{\pi}{l}\right)^6-R\left(\frac{\pi}{l}\right)^4N_{10};$$
$$a_0=\frac{1}{n_{11}}\left[D+\frac{h^3}{12}R(A_{11}-A^*_{11})\left(\frac{\pi}{l}\right)^6-RN_{10}\left(\frac{\pi}{l}\right)^4\right];$$
$$D=\frac{\alpha_{22}a_{22}n_{11}}{n_{11}(a^*_{22}+a_{22})-\alpha_{22}a_{22}}\left[\frac{h^3}{12}\frac{R}{n_{11}}(A_{11}-A^*_{11})\left(\frac{\pi}{l}\right)^6+\right.$$
$$\left.+\frac{h}{R\alpha_{22}a_{22}}\left(\frac{\pi}{l}\right)^2-\frac{R}{n_{11}}\left(\frac{\pi}{l}\right)^4N_{10}\right].$$

The general solution of equation (5.4.11) will be

$$f=C_1e^{r_1t}+C_2e^{r_2t}, \qquad (5.4.12)$$

where $r_{1,2}$ are roots of the characteristic equation

$$a_2r^2+a_1r+a_0=0. \qquad (5.4.13)$$

C_1 and C_2 are determined by random initial sagging/deflection and its speed. So that the amplitude f of random sagging/deflection in the course of time would vanish, it is necessary that all roots of characteristic equation (5.4.13) would have negative real parts.

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Whole root of equation $a_0 + a_1r + a_2r^2 + \ldots + a_nr^n=0$, according to the criterion of Gurits, will have negative real parts only if all determinants

$$D_1=a_1, \quad D_2=\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}, \quad D_3=\begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix}\ldots$$

are positive. For (5.4.13) this condition will take form $a_0 > 0$, $a_1 > 0$, $a_1a_2 > 0$ or last/latter, since $a_1 > 0$ it is possible to replace with condition $a_2 > 0$.

From condition $a_2 > 0$, we obtain the instantaneous critical force (which is greater than prolonged):

$$N_{10} < D_{11}\left(\frac{\pi m}{l}\right)^2+\frac{h}{R^2a_{22}}\left(\frac{l}{m\pi}\right)^2=N_{10}{}^{M}, \quad D_{11}=A_{11}\frac{h^3}{12}. \qquad (5.4.14)$$

After comparing coefficients a_1 and a_0 and taking into account that $A_{11}^{Д}=A_{11}-A^*_{11}<A_{11}$, we are convinced of inequality $a_1 > a_0$. Consequently, it suffices to examine condition $a_0 > 0$. From the latter we obtain the limit of the prolonged stability

$$N_{10} < \frac{h^3}{12} A_{11}^{Д} \frac{\lambda^2}{R^2} + \frac{h}{a_{22}^{Д}} \frac{1}{\lambda^2} = N_{10}^{Д}, \qquad (5.4.15)$$

where

$$A_{11}^{Д}=A_{11}-A^*_{11}; \quad a_{22}^{Д}=a^*_{22}+a_{22}; \quad \lambda=\frac{m\pi R}{l}.$$

Thus, if the applied to shell effort/force N_{10} is less than the limit of prolonged stability $N_{10}^{Д}$, that state of shell stable, since the low random initial sagging/deflection in the course of time disappears.

For obtaining the resultant expression of prolonged critical load (5.4.15) one should minimize λ. After minimization for λ^2 we have

$$\lambda^2=\sqrt{\frac{R^2 h}{D_{11}^{Д} a_{2222}}}; \quad D_{11}^{Д}=A_{1111}^{Д}\frac{h^3}{12}. \qquad (5.4.16)$$

§5.5. Special feature/peculiarities of the mechanical properties of glass-fiber-reinforced plastics and their effect to choose from of kinematic model during the calculation of plates and shells.

The examined, until now, problems are solved with the use of a hypothesis of undeformable standards; however, the limits of the applicability of this hypothesis in the case of anisotropic materials must be establish/installed especially.

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By the basic special feature/peculiarity reinforced plastic is the anisotropy of their deformation properties with low shift cruelty both in the plane of layers of the reinforcing fabric and between layers of fabric. Because of this the standard of cell/element in the process of bending does not remain rectilinear, but curves. As shown in some works [175, 186], for real constructions made of oriented fiber glass plastics of the ratio of Young's modulus to shear modulus they can be such, that even for engineering calculations failure of the hypothesis of the undeformable standards becomes necessary.

Direct measurement of the bending of the cross sections of rods from the oriented glass-fiber-reinforced plastics with elongation and bending is carried out in work [175], where on the basis of experiments is shown the need for the calculation of rods from these materials on the refined formulas without the use of the simplifying hypothesis of flat/plane section/cuts.

Recently in Soviet and foreign authors's works, is given much attention to the explanation of the real character of the stress-strain state in shells and plastics without the use of the simplifying hypothesis. Kirchhoff- Love. One Of the approaches in this direction consists in the application/use of strict mathematical methods [216-219], by the second is based on the construction of the two-dimensional theories of shells and plates during the use of kinematic hypotheses, less rigid, than Kirchhoff-Love's classical hypotheses. By this method it is possible to obtain not the too complex equations, which yield to working and resolution in concrete/specific/actual engineer missions.

The first successful kinematic model, coming out beyond the framework of classical hypotheses, was shear model, S. P. Timoshenko's used as early as in 1921 to the task of transverse

vibrations of beams [299]. According to this model, the standard in the process of deformation remains rectilinear, but normal to deformed median surface.

Important space toward the generalization of the classical theory of the bending of plates was conducted in 1944 by Ye. Reyssner [300], who, on the basis of the semi-inverse method of bringing, assigns the linear law of distribution according to the thickness of the normal part of the stress tensor.

The possible generalization of Ye. Reysssner's works, is shown in A. L. Gol'denveyzer's articles [220] and L. Ya. Aynola [221].

Other theories of the bending of plates, which consider shearing strains, were pushed forward by B. F. Vlasov [222], G. Genkya [301], by A. Kromm [302], P. Nakhdi [303] et al.

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S. A. Ambartsumyan [223-225] during the construction of the generalized theories of anisotropic plates and slightly curved shells also proceeds from the semi-inverse method of bringing, however, unlike Ye. Reyssner, it will assign the law of a change in shearing stresses in thickness.

The further development of the variation of the theory of shells of Timoshenko's type we find in M. P. Sheremetyevo Airport's works [226], 1. Ya. Aynola [227] and others. I placed question about obtaining within the framework of this theory of the relationship/ratios of the joint of deformations. The final version of these relationship/ratios is given in [228, 229].

Subsequently during the investigation of the effect of lateral flexures on the stability of plates and shells, we utilize the kinematic model of Timoshenko's type, developed in articles [230-232] as sufficiently acceptable for application/use in engineering calculations.

During the analysis of stability of plates, we will use also the developed in [231] unified model of Timoshenko's type, which makes it possible to satisfy some conditions on the boundary planes of plate/slab $z = \pm h$.

§5.6. Geometric and static relationship/ratios for shells with the use of a kinematic model of Timoshenko's type.

Let us examine shell by the thickness 2h whose median surface is assigned/prescribed by equation $r = r(\alpha, \beta)$, where α, β are Gaussian coordinates. To each point of median surface $M(\alpha, \beta)$ corresponds trihedron of unit vectors $\tau_{(\alpha)}, \tau_{(\beta)}$ — tangents to α and β lines, n — the vector of standard. Let us accept the following kinematic model of the deformation of the shell: let us consider that the rectilinear filaments of shell, normal to median surface up to deformation, remain rectilinear, also, in the process of deformation, without experience/testing over their entire length of elongation and compressions, but they do not remain perpendicular to deformed median surface. This hypothesis makes it possible to consider transverse shift/shears which is necessary for glass-plastic shells in connection with their low shift rigidity. First this hypothesis is proposed in S. P. Timoshenko's works [299] and later it is developed by other authors [227-232, 300-303].

On the basis of the taken hypothesis of the component of strain

of shell on equidistant surface, are connected by components of strain in median surface by the relationship/ratios:

$$e^*_{\alpha\alpha}=e_{\alpha\alpha}+z\varkappa_1;\quad e^*_{\beta\beta}=e_{\beta\beta}+z\varkappa_2;$$
$$e^*_{\alpha\beta}=e_{\alpha\beta}+z\tau_1+z\tau_2;\quad e^*_{\alpha n}=e_{\alpha n};\quad e^*_{\beta n}=e_{\beta n}, \tag{5.6.1}$$

where

$$\varkappa_1=\frac{1}{A}\frac{\partial\gamma_x}{\partial\alpha}+\frac{1}{AB}\frac{\partial A}{\partial\beta}\gamma_y;$$
$$\varkappa_2=\frac{1}{B}\frac{\partial\gamma_y}{\partial\beta}+\frac{1}{AB}\frac{\partial B}{\partial\alpha}\gamma_x;$$
$$\tau_1=-\frac{\gamma_x}{AB}\frac{\partial A}{\partial\beta}+\frac{1}{A}\frac{\partial\gamma_y}{\partial\alpha};$$
$$\tau_2=-\frac{\gamma_y}{AB}\frac{\partial B}{\partial\alpha}+\frac{1}{B}\frac{\partial\gamma_x}{\partial\beta};$$
$$e_{\alpha n}=\gamma_x+\frac{1}{A}\frac{\partial w}{\partial\alpha};$$
$$e_{\beta n}=\gamma_y+\frac{1}{B}\frac{\partial w}{\partial\beta}.$$

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Here γ_x and γ_y — are angles of rotation of normal filament in planes αn and βn.

For strain in median surface, we have in the case of the tasks of stability the following equation of the consistency of the strains:

$$AB\left(\frac{\varkappa_1}{R_2}+\frac{\varkappa_2}{R_1}\right)=AB\left[\frac{1}{R_2}\left(\frac{1}{A}\frac{\partial e_{\alpha n}}{\partial\alpha}+\frac{e_{\beta n}}{AB}\frac{\partial A}{\partial\beta}\right)+\right.$$
$$\left.+\frac{1}{R_1}\left(\frac{1}{B}\frac{\partial e_{\beta n}}{\partial\beta}+\frac{e_{\alpha n}}{AB}\frac{\partial B}{\partial\alpha}\right)\right]+\frac{\partial^2 e_{\alpha\beta}}{\partial\alpha\partial\beta}+\frac{\partial}{\partial\alpha}\left[\frac{e_{\alpha\beta}}{A}\frac{\partial A}{\partial\beta}+\right.$$
$$\left.+\frac{e_{\alpha\alpha}}{A}\frac{\partial B}{\partial\alpha}-\frac{1}{A}\frac{\partial(Be_{\beta\beta})}{\partial\alpha}\right]+\frac{\partial}{\partial\beta}\left[\frac{e_{\alpha\beta}}{B}\frac{\partial B}{\partial\alpha}+\right.$$
$$\left.+\frac{e_{\beta\beta}}{B}\frac{\partial A}{\partial\beta}-\frac{1}{B}\frac{\partial(Ae_{\alpha\alpha})}{\partial B}\right]. \tag{5.6.2}$$

The equations of equilibrium take the common form. After the examination of the case of moment-less basic state (transverse load is absent), we have the following relationship/ratios:

$$\frac{\partial BN_1}{\partial\alpha}-\frac{\partial B}{\partial\alpha}N_2+\frac{\partial AS}{\partial\beta}+\frac{\partial A}{\partial\beta}S=0; \qquad (5.6.3)$$

$$\frac{\partial AN_1}{\partial\beta}-\frac{\partial A}{\partial\beta}N_1+\frac{\partial BS}{\partial\alpha}+\frac{\partial B}{\partial\alpha}S=0;$$

$$-(k_1N_1+k_2N_2)+\frac{1}{AB}\left(\frac{\partial BQ_1}{\partial\alpha}+\frac{\partial AQ_2}{\partial\beta}\right)=$$

$$=-(N_{10}\bar{\varkappa}_1+N_{20}\bar{\varkappa}_2+2S_0\bar{\chi});$$

$$\frac{\partial BM_1}{\partial\alpha}+\frac{\partial AH}{\partial\beta}+\frac{\partial A}{\partial\beta}H-\frac{\partial B}{\partial\alpha}M_2=ABQ_1;$$

$$\frac{\partial AM_2}{\partial\beta}+\frac{\partial BH}{\partial\alpha}+\frac{\partial B}{\partial\alpha}H-\frac{\partial A}{\partial\beta}M_1=ABQ_2.$$

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Until now, are given the relationship/ratics of the linear theory of shells, which make it possible to investigate stability "in

low". The complete solution of the problem of stability, which involves approach to it "in large", can be given, as is known, only from the positions of nonlinear theory.

Let us give the fundamental principles of theory of Timoshenko's type, which relate to the shells of large sagging/deflection. Let us proceed from the simplified version in which the shell is considered slanting, at least within the limits of separate dent. Let us accept therefore A = B = 1. The initial curvatures of median surface let us designate through $k_1 = 1/R_1$ and $k_2 = 1/R_2$ and accept them for this section of shell as constants.

Let us relate median surface of shell to orthogonal Gaussian coordinates α and β. For strain in equidistant and middle surfaces for the taken model, we have following relationship/ratios [226, 229]:

$$\begin{gathered} e_{\alpha\alpha}=\varepsilon_{11}+z\varkappa_1; \quad e_{\beta\beta}=\varepsilon_{22}+z\varkappa_2; \\ e_{\alpha\beta}=\varepsilon_{12}+2z\varkappa_{12}; \quad e_{\alpha n}=\varepsilon_{13}, \quad e_{\beta n}=\varepsilon_{23}, \end{gathered} \qquad (5.6.4)$$

where

$$\begin{gathered} \varepsilon_{11}=\frac{\partial u}{\partial\alpha}+k_1w+\frac{1}{2}\left(\frac{\partial w}{\partial\alpha}\right)^2, \\ \varepsilon_{22}=\frac{\partial v}{\partial\beta}+k_2w+\frac{1}{2}\left(\frac{\partial w}{\partial\beta}\right)^2, \end{gathered}$$

- $\varepsilon_{12}=\frac{\partial u}{\partial \beta}+\frac{\partial v}{\partial \alpha}+\frac{\partial w}{\partial \alpha}\frac{\partial w}{\partial \beta}$ — tangential strains; $\varepsilon_{13}=\gamma_\alpha+\frac{\partial w}{\partial \alpha}$, $\varepsilon_{23}=\gamma_\beta+\frac{\partial w}{\partial \beta}$ — shearing strain; $\varkappa_1=\frac{\partial \gamma_\alpha}{\partial \alpha}$, $\varkappa_2=\frac{\partial \gamma_\beta}{\partial \beta}$, $\varkappa_{12}=\frac{\partial \gamma_\alpha}{\partial \beta}+\frac{\partial \gamma_\beta}{\partial \alpha}$ — the flexural strains of median surface; u, v, w - the displacement of the points of median surface in directions α, β, z, correspondingly, a γ_α and γ_β — the angles of rotation of standard.

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For by strain in median surface are known four equations of joint [229] from which for the tasks of stability it is substantial one:

$$k_1\varkappa_2+k_2\varkappa_1-\frac{\partial^2\varepsilon_{12}}{\partial\alpha\partial\beta}+\frac{\partial^2\varepsilon_{11}}{\partial\beta^2}+\frac{\partial^2\varepsilon_{22}}{\partial\alpha^2}-k_1\frac{\partial\varepsilon_{23}}{\partial\beta}-k_2\frac{\partial\varepsilon_{13}}{\partial\alpha}=$$
$$=-\frac{1}{2}L(w,w), \qquad (5.6.5)$$

where

$$L(w,w)=2\left[\frac{\partial^2 w}{\partial\alpha^2}\frac{\partial^2 w}{\partial\beta^2}-\left(\frac{\partial^2 w}{\partial\alpha\partial\beta}\right)^2\right];$$
$$k_1=\frac{1}{R_1},\quad k_2=\frac{1}{R_2}.$$

Examining glass-fiber-reinforced plastic as uniform, orthotropic material and disregarding its creep in the direction of reinforcement, we have following physical relationship/ratios (5.4.2):

$$\sigma_{11}=A_{1111}e_{11}+A_{1122}e_{22}; \quad \sigma_{22}A_{2222}e_{22}+A_{2211}e_{11};$$

$$\tau_{12}=A_{1212}e_{12}-\int_0^t R_{1212}(t-\theta)\,e_{12}(\theta)\,d\theta;$$

$$\tau_{13}=A_{1313}e_{13}-\int_0^t R_{1313}(t-\theta)\,e_{13}(\theta)\,d\theta; \qquad (5.6.6)$$

$$\tau_{23}=A_{2323}e_{23}-\int_0^t R_{2323}(t-\theta)\,e_{23}(\theta)\,d\theta.$$

After integration (5.6.6) for the thickness of shell we obtain following expressions for effort/forces and the torque/moments:

$$T_1=2h\,(A_{1111}e_{11}+A_{1122}e_{22});$$

$$T_2=2h\,(A_{2222}e_{22}+A_{2211}e_{11}); \qquad (5.6.7)$$

$$S=2hA_{1212}e_{12}-2h\int_0^t R_{1212}(t-\theta)\,e_{12}(\theta)\,d\theta;$$

$$M_1=\frac{2h^3}{3}\left(A_{1111}\frac{\partial\gamma_\alpha}{\partial\alpha}+A_{2211}\frac{\partial\gamma_\beta}{\partial\beta}\right);$$

$$(5.6.8)$$

$$M_2=\frac{2h^3}{3}\left(A_{2222}\frac{\partial\gamma_\beta}{\partial\beta}+A_{1111}\frac{\partial\gamma_\alpha}{\partial\alpha}\right);$$

$$H=A_{1212}\frac{2h^3}{3}\left(\frac{\partial\gamma_\alpha}{\partial\beta}+\frac{\partial\gamma_\beta}{\partial\alpha}\right)-\frac{2h^3}{3}\int_0^t R_{1212}(t-\theta)\left(\frac{\partial\gamma_\alpha}{\partial\beta}+\frac{\partial\gamma_\beta}{\partial\alpha}\right)d\theta;$$

$$Q_1=A_{1313}2h\left(\gamma_\alpha+\frac{\partial w}{\partial\alpha}\right)-2h\int_0^t R_{1313}(t-\theta)\left(\gamma_\alpha+\frac{\partial w}{\partial\alpha}\right)d\theta; \quad (5.6.9)$$

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$$Q_2=A_{2323}2h\left(\gamma_\beta+\frac{\partial w}{\partial\beta}\right)-2h\int_0^t R_{2323}(t-\theta)\left(\gamma_\beta+\frac{\partial w}{\partial\beta}\right)d\theta.$$

Effort/forces and torque/moments are must satisfy the equations

of the equilibrium:

$$\frac{\partial T_1}{\partial\alpha}+\frac{\partial S}{\partial\beta}=0;\quad \frac{\partial T_2}{\partial\beta}+\frac{\partial S}{\partial\alpha}=0;$$
$$\left(k_1+\frac{\partial^2 w}{\partial\alpha^2}\right)T_1+\left(k_2+\frac{\partial^2 w}{\partial\beta^2}\right)T_2+2S\frac{\partial^2 w}{\partial\alpha\partial\beta}-\frac{\partial Q_1}{\partial\alpha}-\frac{\partial Q_2}{\partial\beta}=$$
$$=T_{10}\frac{\partial^2 w}{\partial\alpha^2}+T_{20}\frac{\partial^2 w}{\partial\beta^2}+2S\frac{\partial^2 w}{\partial\alpha\partial\beta}+q;\qquad (5.6.10)$$
$$\frac{\partial M_1}{\partial\alpha}+\frac{\partial H}{\partial\beta}=Q_1;\quad \frac{\partial M_2}{\partial\beta}+\frac{\partial H}{\partial\alpha}=Q_2,$$

where q - transverse load: T_{10}, T_{20}, S_0 - subcritical moment-less effort/forces.

As usual, let us introduce the function of the voltage/stresses:

$$T_1=\frac{\partial^2 F}{\partial\beta^2};\quad T_2=\frac{\partial^2 F}{\partial\alpha^2};\quad S=\frac{\partial^2 F}{\partial\alpha\partial\beta}.\qquad (5.6.11)$$

System of equations (5.6.10) and the equation of consistency (5.6.5) then with the use of relationship/ratics (5.6.6)-(5.6.9) is reduced to the following final form:

$$\left(k_1+\frac{\partial^2 w}{\partial\alpha^2}\right)\frac{\partial^2 F}{\partial\beta^2}+\left(k_2+\frac{\partial^2 w}{\partial\beta^2}\right)\frac{\partial^2 F}{\partial\alpha^2}-2\frac{\partial^2 F}{\partial\alpha\partial\beta}\frac{\partial^2 w}{\partial\alpha\partial\beta}-A_{1313}2h\times$$
$$\times\left(\frac{\partial\gamma_\alpha}{\partial\alpha}+\frac{\partial^2 w}{\partial\alpha^2}\right)+2h\int_0^t R_{1313}(t-\theta)\left(\frac{\partial\gamma_\alpha}{\partial\alpha}+\frac{\partial^2 w}{\partial\alpha^2}\right)d\theta-$$

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$$-A_{2323}2h\left(\frac{\partial\gamma_\beta}{\partial\beta}+\frac{\partial^2 w}{\partial\beta^2}\right)+2h\int\limits_0^t R_{2323}(t-\theta)\left(\frac{\partial\gamma_\beta}{\partial\beta}+\frac{\partial^2 w}{\partial\beta^2}\right)=$$

$$=T_{10}\frac{\partial^2 w}{\partial\alpha^2}+T_{20}\frac{\partial^2 w}{\partial\beta^2}+2S_0\frac{\partial^2 w}{\partial\alpha\partial\beta}+q;$$

$$\frac{2h}{3}\left(A_{1111}\frac{\partial^2\gamma_\alpha}{\partial\alpha^2}+A_{2211}\frac{\partial^2\gamma_\beta}{\partial\alpha\partial\beta}\right)+A_{1212}\frac{2h^3}{3}\left(\frac{\partial^2\gamma_\alpha}{\partial\beta^2}+\frac{\partial^2\gamma_\beta}{\partial\alpha\partial\beta}\right)-$$

(5.6.12,a, b, c, d)

$$-\frac{2h^3}{3}\int\limits_0^t R_{1313}(t-\theta)\left(\frac{\partial^2\gamma_\alpha}{\partial\beta^2}+\frac{\partial^2\gamma_\beta}{\partial\alpha\partial\beta}\right)d\theta=A_{1313}2h\left(\gamma_\alpha+\frac{\partial w}{\partial\alpha}\right)-$$

$$-2h\int\limits_0^t R_{1313}(t-\theta)\left(\gamma_\alpha+\frac{\partial w}{\partial\alpha}\right)d\theta;$$

$$\frac{2h^3}{3}\left(A_{2222}\frac{\partial^2\gamma_\beta}{\partial\beta^2}+A_{1111}\frac{\partial^2\gamma_\beta}{\partial\alpha\partial\beta}\right)+A_{1212}\frac{2h^3}{3}\left(\frac{\partial^2\gamma_\alpha}{\partial\alpha\partial\beta}+\frac{\partial^2\gamma_\beta}{\partial\alpha^2}\right)-$$

$$-\frac{2h^3}{3}\int\limits_0^t R_{1212}(t-\theta)\left(\frac{\partial^2\gamma_\alpha}{\partial\alpha\partial\beta}+\frac{\partial^2\gamma_\beta}{\partial\alpha^2}\right)d\theta=A_{2323}2h\left(\gamma_\beta+\frac{\partial w}{\partial\beta}\right)-$$

$$-2h\int\limits_0^t R_{2323}(t-\theta)\left(\gamma_\nu+\frac{\partial w}{\partial\beta}\right)d\theta;$$

$$a_{1111}\frac{1}{2h}\frac{\partial^4 F}{\partial\beta^4}+a_{2222}\frac{1}{2h}\frac{\partial^4 F}{\partial\alpha^4}+(2a_{2211}+a_{1212})\frac{1}{2h}\frac{\partial^4 F}{\partial\alpha^2\partial\beta^2}-$$

$$-k_1\frac{\partial^2 w}{\partial\beta^2}-k_2\frac{\partial^2 w}{\partial\alpha^2}+\frac{1}{2h}\int\limits_0^t K_{1212}(t-\theta)\frac{\partial^4 F}{\partial\alpha^2\partial\beta^2}d\theta=$$

$$=-\frac{1}{2}L(w,w).$$

Disregarding in (5.6.12) nonlinear terms, we obtain the matching system of the linear equations of the stability theory of anisotropic shells taking into account the strain of transverse shift/shears.

The nonlinear and linear resolving systems of the stability of plates we will obtain respectively with $k_1 = k_2 = 0$.

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§5.7. Relationship/ratios of the stability of orthotropic plates on the base of the generalization of Timoshenko's kinematic model.

Let us generalize the obtained above results and use to stability analysis the theory of plates, which makes it possible to satisfy some conditions on the boundary planes of plate/slab. Following [231], let us record the basic kinematic relationship/ratios of the theory of plates, which considers not only rotation, but also the bending of standard.

Let us examine plate by the thickness 2h whose medium plane is referred to the orthogonal system of coordinates x, y. Let us assume that the filament, normal to medium plane up to strain, in the process of deformation is turned and is bent, without experience/testing shortening and elongation (i.e. $\varepsilon_z=0$); normal stresses σ_z are considered low in comparison with other stresses. In this field of the displacement of plate/slab, we determine by formulas [231]:

$$\begin{aligned} u &= u^0+z\gamma_x{}^0+z^2u^T+z^3\gamma_x{}^T; \\ v &= v^0+z\gamma_y{}^0+z^2v^T+z^3\gamma_y{}^T; \qquad (5.7.1) \\ w &= w^0. \end{aligned}$$

Here γ_x^0 and γ_y^0 — are angles of rotation of filament in planes xz and uz at the level of median surface. If we the indicated dependences are restricted linear relative to z to terms, we will obtain the variation of the theory of S. P. Timoshenko's type [231], according to whom it is accepted that the normal filament in the process of deformation remains rectilinear, but normal to median surface. Coefficients $u^T, v^T, \gamma_x^T, \dot{\gamma}_y^T$ let us determine from boundary conditions [231].

By satisfaction to boundary conditions for shearing stresses

$$\tau_{xz}|_{z=\pm h}=\tau_{xz}^{\pm} \quad \tau_{yz}|_{z=\pm h}=\tau_{yz}^{\pm} \tag{5.7.2}$$

we obtain following convenient formula for coefficients indicated above in the case when shearing stresses on the lateral surfaces of plate are equal to zero [231]:

$$\gamma_x^T=-\frac{1}{3h^2}\left(\gamma_x^{(0)}+\frac{\partial w_0}{\partial x}\right), \quad u^T=0;$$

$$\gamma_y^T=-\frac{1}{3h^2}\left(\gamma_y^{(0)}+\frac{\partial w^0}{\partial y}\right), \quad v^T=0. \tag{5.7.3}$$

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In work [231] are obtained the expressions of the indicated coefficients also for the case of assignment on the boundary planes of the tangential components of displacement, also mixed boundary conditions. However, these expressions subsequently we will not use.

The given formulas show that, unlike the classical theory of plates. Kirchhoff, here the tangential displacements u, v of the arbitrary point of plate, removed from medium plane along the normal up to distance z, generally depend on z nonlinear. We note also that unlike the classical theory of the thin plates here the attachment of all points of middle plane (for the target/purpose of the non-admission of any displacements and rotations in space) does not lead plate to the state of undeformable body.

Utilizing the obtained above expressions for displacements we obtain the relationship/ratios of strain in the equidistant plane:

$$e_{xx}=\varepsilon_{11}+z\varkappa_1-\frac{z^3}{3h^2}\left(\frac{\partial\gamma_x}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right);$$

$$e_{yy}=\varepsilon_{22}+z\varkappa_2-\frac{z^3}{3h^2}\left(\frac{\partial\gamma_y}{\partial y}+\frac{\partial^2 w}{\partial y^2}\right); \qquad (5.7.4)$$

$$e_{xy}=\varepsilon_{12}+2z\varkappa_{12}-\frac{z^3}{3h^2}\left(\frac{\partial\gamma_x}{\partial y}+\frac{\partial\gamma_y}{\partial x}+2\frac{\partial^2 w}{\partial x\partial y}\right);$$

$$e_{xz}=\varepsilon_{13}\left(1-\frac{z^2}{h^2}\right);$$

$$e_{yz}=\varepsilon_{23}\left(1-\frac{z^2}{h^2}\right),$$

where

$$\varepsilon_{11}=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2;$$

$$\varepsilon_{22}=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2;$$

$$\varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y};$$

$$\varepsilon_{13}=\gamma_x+\frac{\partial w}{\partial x};$$

$$\varepsilon_{23}=\gamma_y+\frac{\partial w}{\partial y};$$

$$\varkappa_1=\frac{\partial\gamma_x}{\partial x};\quad \varkappa_2=\frac{\partial\gamma_y}{\partial y};\quad \varkappa_{12}=\frac{\partial\gamma_x}{\partial x}+\frac{\partial\gamma_y}{\partial y}.$$

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On the strength of assumption $\varepsilon_z=\frac{\partial w}{\partial z}=0$ we will obtain

$$w = w(x, y). \qquad (5.7.5)$$

i.e. just as in the classical theory of plates, the displacement of any point of plate in the direction z does not depend on z and for all points of this normal cell/element equal to the normal displacement w at the appropriate point of medium plane.

As physical relationship/ratios let us accept expressions for an orthotropic material with rheonomic properties. Will be here taken into account as earlier, only shift creep of interlayer shift/shear and shift/shear in the plane of layers of the reinforcing fabric. In this case, it is accepted that in the fiber direction creep from normal stresses is absent (5.2.4).

Calculating the values of internal effort/forces, during use (5.2.4) we will have:

$$M_x = \frac{4}{5}\left(D_x \frac{\partial \gamma_x}{\partial x} + D_1 \frac{\partial \gamma_y}{\partial y}\right) - \frac{1}{5}\left(D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2}\right);$$

$$M_y = \frac{4}{5}\left(D_y \frac{\partial \gamma_y}{\partial y} + D_1 \frac{\partial \gamma_x}{\partial x}\right) - \frac{1}{5}\left(D_y \frac{\partial^2 w}{\partial y^2} + D_1 \frac{\partial^2 w}{\partial x^2}\right);$$

$$H_{xy} = \frac{4}{5} D_{xy}\left(\frac{\partial \gamma_y}{\partial x} + \frac{\partial \gamma_x}{\partial y}\right) - \frac{2}{5} D_{xy} \frac{\partial^2 w}{\partial x \partial y} -$$

$$- \frac{4}{5} B \int_0^t \bar{G}_{12}(t-\theta)\left(\frac{\partial \gamma_x}{\partial y} + \frac{\partial \gamma_y}{\partial x} - \frac{1}{2}\frac{\partial^2 w}{\partial x \partial y}\right) d\theta; \qquad (5.7.6)$$

$$Q_x = \frac{4}{3} G_{13} h\left(\gamma_x + \frac{\partial w}{\partial x}\right) - \frac{4}{3} h \int_0^t \bar{G}_{13}(t-\theta)\left(\gamma_x + \frac{\partial w}{\partial x}\right) d\theta;$$

$$Q_y = \frac{4}{3} G_{23} h\left(\gamma_y + \frac{\partial w}{\partial y}\right) - \frac{4}{3} h \int_0^t \bar{G}_{23}(t-\theta)\left(\gamma_y + \frac{\partial w}{\partial y}\right) d\theta.$$

Here

$$D_x=\frac{2A_{1111}h^3}{3};\quad D_y=\frac{2A_{2222}h^3}{3};\quad D_{xy}=\frac{2G_{1212}h^3}{3};\quad D_1=\frac{2A_{1122}h^3}{3};$$
$$B=\frac{2}{3}h^3.$$

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Utilizing further an equation of equilibrium (5.610) and of consistency (5.6.5) with $k_1 = k_2 = 0$ and introducing the function of stress F by formulas (5.6.11), we will obtain in summation, the following resolving equations of task:

$$\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2}-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y}-$$

$$-A_{1313}\frac{4}{3}h\left(\frac{\partial\gamma_x}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)-$$

$$-\frac{4}{3}h\int_0^t R_{1313}(t-\theta)\left(\frac{\partial\gamma_x}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)d\theta-A_{2323}\frac{4}{3}h\left(\frac{\partial\gamma_y}{\partial y}+\right.$$

$$\left.+\frac{\partial^2 w}{\partial y^2}\right)-\frac{4}{3}h\int_0^t R_{2323}(t-\theta)\left(\frac{\partial\gamma_y}{\partial y}+\frac{\partial^2 w}{\partial y^2}\right)d\theta=$$

$$=T_{10}\frac{\partial^2 w}{\partial x^2}+T_{20}\frac{\partial^2 w}{\partial y^2}+2S_0\frac{\partial^2 w}{\partial x\partial y}+.q;$$

$$\frac{2h^3}{3}\left\{\left[\frac{4}{5}\left(A_{1111}\frac{\partial^2\gamma_x}{\partial x^2}+A_{1212}\frac{\partial^2\gamma_x}{\partial y^2}\right)+(A_{2211}+A_{1212})\frac{\partial^2\gamma_y}{\partial x\partial y}\right]-\right.$$

$$\left.-\frac{1}{5}\left[A_{1111}\frac{\partial^3 w}{\partial x^3}+(2A_{1212}+A_{2211})\frac{\partial^3 w}{\partial x\partial y^2}\right]\right\}-$$

$$-\frac{2h^3}{3}\int_0^t R_{1212}(t-\theta)\left[\frac{4}{5}\left(\frac{\partial^2\gamma_x}{\partial y^2}+\frac{\partial^2\gamma_y}{\partial x\partial y}\right)-\frac{2}{5}\frac{\partial^3 w}{\partial x\partial y^2}\right]d\theta= \tag{5.7.7}$$

$$=A_{1313}\frac{4}{3}h\left(\gamma_x+\frac{\partial w}{\partial x}\right)-\frac{4}{3}h\int_0^t R_{1313}(t-\theta)\left(\gamma_x+\frac{\partial w}{\partial x}\right)d\theta;$$

$$\frac{2h^3}{3}\left\{\left[\frac{4}{5}\left(A_{2222}\frac{\partial^2\gamma_y}{\partial y^2}+A_{1212}\frac{\partial^2\gamma_y}{\partial x^2}\right)+(A_{1212}+A_{2211})\frac{\partial^2\gamma_x}{\partial x\partial y}\right]-\right.$$

$$\left.-\frac{1}{5}\left[A_{2222}\frac{\partial^3 w}{\partial y^3}+(2A_{1212}+A_{2211})\frac{\partial^3 w}{\partial x^2\partial y}\right]\right\}-\frac{2h^3}{3}\int_0^t R_{1212}(t-$$

$$-\theta)\left[\frac{4}{5}\left(\frac{\partial^2\gamma_x}{\partial x\partial y}+\frac{\partial^2\gamma_y}{\partial x^2}\right)-\frac{2}{5}\frac{\partial^3 w}{\partial x^2\partial y}\right]d\theta=$$

$$=A_{2323}\frac{4}{3}h\left(\gamma_y+\frac{\partial w}{\partial y}\right)-\frac{4}{3}h\int_0^t R_{2323}(t-\theta)\left(\gamma_y+\frac{\partial w}{\partial y}\right)d\theta;$$

$$a_{1111}\frac{\partial^4 F}{\partial y^4}+a_{2222}\frac{\partial^4 F}{\partial x^4}+(2a_{2211}+a_{1212})\frac{\partial^4 F}{\partial x^2\partial y^2}+$$

$$+\frac{1}{2h}\int_0^t K_{1212}(t-\theta)\frac{\partial^4 F}{\partial x^2\partial y^2}d\theta=-\frac{1}{2}L(w,w).$$

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equations (5.7.7) compose the complete resolving system of four integrodifferential equations of relatively three unknown functions F: w, γ_x, γ_y.

For the linear case, eliminating nonlinear terms, we will obtain relative to three functions w, γ_x, γ_y the following system:

$$G_{13}{}^0\frac{\partial^2 w}{\partial x^2}+G_{23}{}^0\frac{\partial^2 w}{\partial y^2}+G_{13}{}^0\frac{\partial\gamma_x}{\partial x}+G_{23}{}^0\frac{\partial\gamma_y}{\partial y}-\int_0^t\bar{G}_{13}{}^0(t-$$

$$-\theta)\left(\gamma_x+\frac{\partial w}{\partial x}\right)d\theta-\int_0^t\bar{G}_{23}{}^0(t-\theta)\left(\gamma_y+\frac{\partial w}{\partial y}\right)d\theta=N\frac{\partial^2(w_0+w)}{\partial x^2};$$

$$\frac{4}{5}\left(D_x\frac{\partial^2}{\partial x^2}+D_{xy}\frac{\partial^2}{\partial y^2}\right)\gamma_x+\frac{4}{5}(D_1+D_{xy})\frac{\partial^2\gamma_y}{\partial x\partial y}-\frac{1}{5}\left[D_x\frac{\partial^3}{\partial x^3}+\right.$$

$$\left.+(D_1+2D_{xy})\frac{\partial^3}{\partial x\partial y^2}\right]w-\frac{4}{5}B\int_0^t\bar{G}_{13}(t-\theta)\left(\frac{\partial^2\gamma_x}{\partial y^2}+\frac{\partial^2\gamma_y}{\partial x\partial y}-\right.$$

$$\left.-\frac{1}{2}\frac{\partial^3 w}{\partial x\partial y^2}\right)d\theta-G_{13}{}^0\left(\gamma_x+\frac{\partial w}{\partial x}\right)+\int_0^t\bar{G}_{13}{}^0(t-\theta)\left(\gamma_x+\right.$$

$$\left.+\frac{\partial w}{\partial x}\right)d\theta=0;$$

$$(5.7.8)$$

$$\frac{4}{5}\left(D_y\frac{\partial^2}{\partial y^2}+D_{xy}\frac{\partial^2}{\partial x^2}\right)\gamma_y+\frac{4}{5}(D_1+D_{xy})\frac{\partial^2\gamma_x}{\partial x\partial y}-\frac{1}{5}\left[D_y\frac{\partial^3}{\partial y^3}+\right.$$

$$\left.+(D_1+2D_{xy})\frac{\partial^3}{\partial x^2\partial y}\right]w-\frac{4}{5}B\int_0^t\bar{G}_{12}(t-\theta)\left(\frac{\partial^2\gamma_y}{\partial x^2}+\frac{\partial^2\gamma_x}{\partial x\partial y}-\right.$$

$$\left.-\frac{1}{2}\frac{\partial^3 w}{\partial x^2\partial y}\right)d\theta-G_{23}{}^0\left(\gamma_y+\frac{\partial w}{\partial y}\right)+\int_0^t\bar{G}_{23}{}^0(t-\theta)\left(\gamma_y+\right.$$

$$\left.+\frac{\partial w}{\partial y}\right)d\theta=0.$$

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Here

$$G_{i3}^0=\frac{4}{3}hG_{i3}\quad(i=1,2).$$

Equations (5.7.8) compose the complete resolving system of three integrodifferential equations of the relatively three unknown functions w, γ_x, γ_y, limitedness of which in time determines the stability of plate.

During the solution to concrete/specific/actual boundary-value problems to the resolving integrodifferential equations of plate, one should connect boundary conditions.

For example, in the particular case of hinged support for edges boundary conditions are satisfied, if unknown function with the following form:

$$\begin{aligned} w&=f(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b};\\ \gamma_x&=\chi_1(t)\cos\frac{\pi x}{a}\sin\frac{\pi y}{b};\\ \gamma_y&=\chi_2(t)\sin\frac{\pi x}{a}\cos\frac{\pi y}{b}.\end{aligned}\qquad(5.7.9)$$

Taking into account (5.7.9) system (5.7.8) will take the form

$$a_{11}f(t)+a_{12}\chi_1(t)+a_{13}\chi_2(t)+\frac{\pi}{a}I_{13}+\frac{\pi}{b}I_{23}=-N\left(\frac{\pi}{a}\right)^2(f+f_0);$$

$$a_{21}f(t)+a_{22}\chi_1(t)+a_{23}\chi_2(t)+\frac{4}{5}BI_{12}+I_{13}=0; \qquad (5.7.10)$$

$$a_{31}f(t)+a_{32}\chi_1(t)+a_{33}\chi_2(t)+\frac{4}{5}\frac{b}{a}BI_{12}+I_{23}=0,$$

where

$$I_{13}=\int_0^t \bar{G}_{13}^0(t-\theta)\left[\chi_1(\theta)+\frac{\pi}{a}f(\theta)\right]d\theta;$$

$$I_{23}=\int_0^t \bar{G}_{23}(t-\theta)\left[\chi_2(\theta)+\frac{\pi}{b}f(\theta)\right]d\theta;$$

$$I_{12}=\int_0^t \bar{G}_{12}^0(t-\theta)\left[\left(\frac{\pi}{b}\right)^2\chi_1(\theta)+\frac{\pi^2}{ab}\chi_2(\theta)-\frac{\pi^3}{2ab^2}f(\theta)\right]d\theta;$$

$$\bar{G}_{i3}^0(t-\theta)=\frac{4}{3}h\bar{G}(t-\theta) \qquad (i=1,2);$$

$$a_{11}=-G_{13}^0\left(\frac{\pi}{a}\right)^2-G_{23}^0\left(\frac{\pi}{b}\right)^2; \quad a_{12}=-\frac{\pi}{a}G_{13}^0; \quad a_{13}=-\frac{\pi}{b}G_{23}^0;$$

$$a_{21}=\frac{1}{5}\left[D_x\left(\frac{\pi}{a}\right)^3+(D_1+2D_{xy})\frac{\pi^3}{ab^2}\right]-\frac{\pi}{a}G_{13}^0;$$

$$a_{22}=-\frac{4}{5}\left[D_x\left(\frac{\pi}{a}\right)^2+D_{xy}\left(\frac{\pi}{b}\right)^2\right]-G_{13}^0;$$

$$a_{23}=-\frac{4}{5}(D_1+D_{xy})\frac{\pi^2}{ab};$$

$$a_{31}=\frac{1}{5}\left[D_y\left(\frac{\pi}{b}\right)^3+(D_1+2D_{xy})\frac{\pi^3}{a^2b}\right]-\frac{\pi}{b}G_{23}^0; \quad a_{32}=a_{23};$$

$$a_{33}=-\frac{4}{5}\left[D_y\left(\frac{\pi}{b}\right)^2+D_{xy}\left(\frac{\pi}{a}\right)^2\right]-G_{23}^0.$$

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Thus, we will obtain the system of integral equations. Hereditary function must be accepted in accordance with experiments on the analysis of the deformation properties of material. The stability of plate is determined by limitedness in time f(t), $\chi_1(t)$ and $\chi_2(t)$, that, in particular, can be investigated with the aid of high speed calculators (example of the solution of a similar problem for geometrically nonlinear slightly curved shell will be given below).

If the experimental data of the properties of material are described by exponential nuclei, then the system of integral equations (5.7.10) we convert into the system of the differential equations of the following form:

$$\begin{aligned}\dot{f} &= \alpha_{11}f+\alpha_{12}\chi_1+\alpha_{13}\chi_2;\\ \dot{\chi}_1 &= \alpha_{21}f+\alpha_{22}\chi_1+\alpha_{23}\chi_3;\\ \dot{\chi}_2 &= \alpha_{31}f+\alpha_{32}\chi_1+\alpha_{33}\chi_2.\end{aligned} \tag{5.7.11}$$

where the coefficients α_{ij} by known form are expressed as the constants of the deformation properties of material, the geometric parameters of shell and subcritical effort/forces. For the determination of critical loads to system (5.7.11) are used Hurwitz' criteria.

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§5.8. Cylindrical bulge during creep of orthotropic plate. Limits of the applicability of Kirchhoff-Love's hypothesis.

We investigate the cylindrical bulge of plate during creep taking into account the refined theory of plates, examined into §5.7, according to which the field of the displacement of plate/slab is determined by the dependences:

$$u=u^0+z\gamma_x{}^0+z^2u^T+z^3\gamma_x{}^T;$$
$$v=v^0+z\gamma_y{}^0+z^2v^T+z^3\gamma_y{}^T;\quad w=w^0. \tag{5.8.1}$$

Taking into account the fact that creep of some glass-fiber-reinforced plastics little is developed in the direction of reinforcement, for ε_{11} we utilize Hooke's law, taking into account only shift creep in plane xz (material orthotropic):

$$\sigma_x=\frac{E_1}{1-\nu_1\nu_2}\varepsilon_1=E'\varepsilon_1;\quad \sigma_{xz}=G_{13}\gamma_{xz}-\int_0^t\bar{G}_{13}(t-\theta)\gamma_{xz}d\theta. \tag{5.8.2}$$

For this case the flow properties of glass-plastic plates cannot be described with the aid of the model of Kirchoff according to which in the task in question we obtain only instantaneous Euler critical force $T_э$; the process of loss of stability in time here it can be investigated only with the aid of the refined theory of the bending

of plates.

In order to calculate internal torque M_x and transverse force we utilize dependences for the field of the displacement of plate/slab in the case of the cylindrical bending:

$$u = z\gamma_x^0 + z^3\gamma_x^T; \quad w = w^0, \tag{5.8.3}$$

where

$$\gamma_x^T = -\frac{1}{3h^2}\left(\gamma_x^0 + \frac{\partial w}{\partial x}\right).$$

Taking into account the fact that

$$e_x = \frac{\partial u}{\partial x} = z\frac{\partial \gamma_x^0}{\partial x} + z^3\frac{\partial \gamma_x^T}{\partial x}$$

and

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \gamma_x^0 + 3\gamma_x^T z^2 + \frac{\partial w}{\partial x},$$

let us compute M_x and Q_x:

$$\begin{gathered} M_x = \frac{4}{5}D_x\frac{\partial \gamma_x^0}{\partial x} - \frac{1}{5}D_x\frac{\partial^2 w}{\partial x^2}; \\ Q_x = \frac{4}{3}G_{13}h\left(\gamma_x^0 + \frac{\partial w}{\partial x}\right) - \frac{4}{3}h\int_0^t \bar{G}_{13}(t-\theta)\left(\gamma_x^0 + \frac{\partial w}{\partial x}\right)d\theta, \end{gathered} \tag{5.8.4}$$

where the thickness of plate/slab $2h$, $D_x = \frac{2E'h^3}{3}$.

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The equations of the equilibrium of differential cell/element as the compressed force T_{10} of plate with initial sagging/deflection w_{00} with cylindrical bending has the form

$$\frac{\partial M_x}{\partial x}=Q_x; \quad \frac{\partial Q_x}{\partial x}-T_{10}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w_{00}}{\partial x^2}\right)=0. \qquad (5.8.5)$$

After substituting (5.8.4) in (5.8.5), we have

$$\frac{4}{5}D_x\frac{\partial^3\gamma_x{}^0}{\partial x^3}-\frac{1}{5}D_x\frac{\partial^4 w}{\partial x^4}=T_{10}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w_{00}}{\partial x^2}\right);$$

$$\frac{4}{3}G_{13}h\left(\frac{\partial\gamma_x{}^0}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)-\frac{4}{3}h\int_0^t \overline{G}_{13}(t-\theta)\left(\frac{\partial\gamma_x{}^0}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)d\theta-$$

$$-T_{10}\frac{\partial^2(w_{00}-w)}{\partial x^2}=0. \qquad (5.8.6a,b)$$

In the examination of the hinged support of plate, we take

$$w=f(t)\sin\frac{\pi x}{a}; \quad w_{00}=f_{00}\sin\frac{\pi x}{a};$$

$$\gamma_x{}^0=\chi(t)\cos\frac{\pi x}{a}. \qquad (5.8.7)$$

After accepting $\overline{G}_{13}(t-\theta)$ in the form of the exponential $\overline{G}_{13}(t-\theta)=$ $=\frac{G_{13}-G_{D13}}{n}e^{-\frac{t-\theta}{n}}$, taking into account (5.8.7) and after eliminating from system (5.8.6) $\chi(t)$, for the amplitude of sagging/deflection f(t)

we obtain

$$n\left\{\frac{5}{3}G_{13}h-\left[\frac{5}{3}\frac{G_{13}h}{D_x}\left(\frac{a}{\pi}\right)^2+1\right]T_{10}\right\}\dot{f}+\left\{\frac{5}{3}G_{D13}h-\right.$$
$$\left.-\left[\frac{5}{3}\frac{G_{D13}h}{D_x}\left(\frac{a}{\pi}\right)^2+1\right]T_{10}\right\}f=\left[\frac{5}{3}\frac{G_{D13}h}{D_x}+\left(\frac{\pi}{a}\right)^2\right]T_{10}f_{00}. \tag{5.8.8}$$

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If function f (t) is known, then for computation $\chi(t)$, accordingly (5.8.6a), we have the following linear dependence:

$$\chi(t)=\frac{5}{4}\frac{a}{D_x\pi}\left[\frac{1}{5}D_x\left(\frac{\pi}{a}\right)^2-T_{10}\right]f(t)-\frac{5}{4}\frac{a}{D_x\pi}T_{10}f_{00}. \tag{5.8.9}$$

It is solved (by 5.8.8) under the initial condition

$$f_{t=0}=f_0=\frac{f_{00}}{\frac{T_M}{T}-1}, \tag{5.8.10}$$

where $T_M=\frac{T_э}{1+\frac{2}{5}\frac{Eh^2}{G_{13}}\left(\frac{\pi}{a}\right)^2}$ — the critical load of elastic task taking into account transverse shift/shears. Here $T_э=D_x\left(\frac{\pi}{a}\right)^2$ — critical load for the plate in question without taking into account of transverse shift/shears (in this case is possible only instantaneous it is elastic loss of stability).

After solving (5.8.8), we have

$$f(t)=f_\infty+(f_0-f_\infty)e^{\frac{T_Д-T_{10}}{T_М-T_{10}}\frac{t}{n}}, \tag{5.8.11}$$

where $f_\infty=\frac{f_{00}}{\frac{T_Д}{T_{10}}-1}$ — sagging/deflection with t = ∞;

$$T_Д=\frac{T_Э}{1+\frac{2}{5}\frac{Eh^2}{G_{Д13}}\left(\frac{\pi}{a}\right)^2}=\frac{T_Э}{1+\frac{3}{5}\frac{T_Э}{G_{Д13}h}}$$ —

- prolonged critical load taking into account transverse shift/shears.

From (5.8.11) it follows that when $T_{10}<T_Д$ the sagging/deflections in time attenuate (steady state), and when $T_Д<T_{10}<T_М$ the rate of the increase of sagging/deflections grow/rises; when $T_{10}=T_М$ we have instantaneous loss of stability.

Let us estimate error appearing as a result of the disregard of shift/shears in plane xz and of the disregard of the creep of the material:

$$\eta=\frac{T_М-T_Э}{T_М}100\%=\frac{2}{5}\left(\frac{h}{a}\right)^2\frac{E'}{G_{13}}\pi^2\,100\%;$$

$$\bar{\eta}=\frac{T_Д-T_Э}{T_Д}100\%=\frac{2}{5}\left(\frac{h}{a}\right)^2\frac{E'}{G_{Д13}}\pi^2\,100\%. \tag{5.8.12}$$

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Computations show that transverse shift/shears one should consider for cell/elements with flexibility to 30. Errors (in o/o), that appear with the disregard of transverse shift/shears, for cell/elements by flexibility to 30 are given in table 5.1.

Table 5.1. Errors from the disregard of transverse shift/shears in o/o.

$\frac{a}{2h}$	$\frac{E'}{G_{13}}$ (1) или $\frac{E'}{G_{D13}}$					
	3	5	10	20	40	60
10	3,0	5,0	10,0	20,0	40,0	60,0
20	0,8	1,3	2,5	5,0	10,0	15,0
30	0,3	0,6	1,1	2,2	4,4	6,7

Key: (1). or.

§5.9. Stability of orthotropic cylindrical shells during creep taking into account the strains of transverse shift/shears.

We investigate the stability of orthotropic shells during linear creep taking into account shift creep and strains of transverse shift/shears. It is assumed that the standard to median surface of shell in the process of strain does not remain perpendicular to it, but it is turned at some angle, without experience/testing elongations - compressions and without bending (kinematic model of Timoshenko's type). This model, as is known, makes it possible to consider shearing strains, not taking into account of the classical theory of kirkhgofa - Love's shells.

1. If we consider glass-fiber-reinforced plastic as orthotropic material and to disregard its creep in direction of reinforcement, then we will obtain physical relationship/ratios from (5.2.4).

According to the taken model of the deformation of shell, we have the following geometric constraints between strains on equidistant and median surfaces:

$$e_{11}=\varepsilon_{11}+z\varkappa_1;\quad e_{22}=\varepsilon_{22}+z\varkappa_2;$$
$$e_{12}=\varepsilon_{12}+2z\varkappa_{12};\quad e_{13}=\varepsilon_{13};\quad e_{23}=\varepsilon_{23},\tag{5.9.1}$$

moreover

$$\varkappa_1=\frac{\partial\gamma_x}{\partial x};\quad \varkappa_2=\frac{\partial\gamma_y}{\partial y};\quad 2\varkappa_{12}=\frac{\partial\gamma_x}{\partial y}+\frac{\partial\gamma_y}{\partial x};\quad \varepsilon_{13}=\gamma_x+\frac{\partial w}{\partial x};$$
$$\varepsilon_{23}=\gamma_y+\frac{\partial w}{\partial y},\tag{5.9.2}$$

where e_{ij} - strain in equidistant surface;

ε_{ij} - strain in median surface;

γ_x, γ_y - the angles of rotation of normal filament in planes xz

and yz.

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For strains in median surface, we have equation of the continuity:

$$\frac{\partial^2 e_{11}}{\partial y^2}+\frac{\partial^2 e_{22}}{\partial x^2}-\frac{\partial^2 e_{12}}{\partial x\partial y}+k_2\varkappa_1+k_1\varkappa_2-k_2\frac{\partial e_{13}}{\partial x}-k_1\frac{\partial e_{23}}{\partial y}=0. \tag{5.9.3}$$

The equations of the equilibrium of the differential cell/element of shell will be:

$$\begin{gathered}\frac{\partial T_1}{\partial x}+\frac{\partial S}{\partial y}=0; \quad \frac{\partial S}{\partial x}+\frac{\partial T_2}{\partial y}=0;\\ \frac{\partial Q_1}{\partial x}+\frac{\partial Q_2}{\partial y}-k_1T_1-k_2T_2-T_{10}\frac{\partial^2 w}{\partial x^2}-T_{20}\frac{\partial^2 w}{\partial y^2}-2S_0\frac{\partial^2 w}{\partial x\partial y}=0;\\ \frac{\partial M_1}{\partial x}+\frac{\partial H}{\partial y}-Q_1=0; \quad \frac{\partial H}{\partial x}+\frac{\partial M_2}{\partial y}-Q_2=0,\end{gathered} \tag{5.9.4}$$

where the expressions of internal effort/forces taking into account (5.2.4) and (5.9.5) take the form

$$T_1=\int_{-h}^{h}\sigma_{11}dz=2h(A_{1111}\varepsilon_{11}+A_{1122}\varepsilon_{22});\quad T_2=\int_{-h}^{h}\sigma_{22}dz=2h(A_{2222}+$$
$$+A_{2211}\varepsilon_{11});\tag{5.9.5}$$
$$S=\int_{-h}^{h}\sigma_{12}dz=2hA_{1212}\varepsilon_{12}-2h\int_0^t R_{1212}(t-\theta)\,\varepsilon_{12}(\theta)\,d\theta;$$
$$M_1=\int_{-h}^{h}\sigma_{11}zdz=\frac{2h^3}{3}\left(A_{1111}\frac{\partial\gamma_x}{\partial x}+A_{2222}\frac{\partial\gamma_y}{\partial y}\right);$$
$$M_2=\int_{-h}^{h}\sigma_{22}zdz=\frac{2h^3}{3}\left(A_{2222}\frac{\partial\gamma_y}{\partial y}+A_{1111}\frac{\partial\gamma_y}{\partial x}\right);$$
$$H=\int_{-h}^{h}\tau_{12}zdz=A_{1212}\frac{2h^3}{3}\left(\frac{\partial\gamma_x}{\partial y}+\frac{\partial\gamma_y}{\partial x}\right)-\frac{2h^3}{3}\int_0^t R_{1212}(t-$$
$$-\theta)\left(\frac{\partial\gamma_x}{\partial y}+\frac{\partial\gamma_y}{\partial x}\right)d\theta;$$
$$Q_1=\int_{-h}^{h}\tau_{13}dz=A_{1313}2h\left(\gamma_x+\frac{\partial w}{\partial x}\right)-2h\int_0^t R_{1313}(t-\theta)\times$$
$$\times\left(\gamma_x+\frac{\partial w}{\partial x}\right)d\theta;$$
$$Q_2=\int_{-h}^{h}\tau_{23}dz=A_{2323}2h\left(\gamma_y+\frac{\partial w}{\partial y}\right)-$$
$$-2h\int_0^t R_{2323}(t-\theta)\left(\gamma_y+\frac{\partial w}{\partial y}\right)d\theta.$$

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Here 2h are thickness of shell.

Following the mixed method and representing the tangential effort/forces T_1, T_2 and S through the function of stresses F, from relationship/ratios (5.9.3) and (5.9.4) we will obtain four fundamental equations relatively unknown of functions F, w, γ_x and γ_y, into which will enter the integral terms:

$$\int_0^t R_{1212}(t-\theta)\left(\frac{\partial\gamma_x}{\partial y}+\frac{\partial\gamma_y}{\partial x}\right)d\theta;\quad \int_0^t R_{1313}(t-\theta)\left(\gamma_x+\frac{\partial w}{\partial x}\right)d\theta;$$

$$\int_0^t R_{2323}(t-\theta)\left(\gamma_y+\frac{\partial w}{\partial y}\right)d\theta. \quad (5.9.6)$$

If we eliminate from the obtained four equations function F, and then to give to shell disturbance/perturbation in the form of low initial deviations (sagging/deflection and angles of rotation), for example in the form

$$w=f\sin\frac{m\pi x}{l}\sin\frac{n\pi y}{l};\quad \gamma_x=\chi_1\cos\frac{m\pi x}{l}\sin\frac{n\pi y}{l};$$
$$\gamma_y=\chi_2\sin\frac{m\pi x}{l}\cos\frac{n\pi y}{l}, \tag{5.9.7}$$

and finally to pass (with exponential nuclei) due to integral equations to differential, we will obtain in summation, for research on the development of strains and process of the bulge of shell in time the system of the form

$$\dot{f}=\alpha_{11}f+\alpha_{12}\chi_1+\alpha_{13}\chi_2;\quad \dot{\chi}_1=\alpha_{21}f+\alpha_{22}\chi_1+\alpha_{23}\chi_2;$$
$$\dot{\chi}_2=\alpha_{31}f+\alpha_{32}\chi_1+\alpha_{33}\chi_2, \tag{5.9.8}$$

where α_{ij} are coefficients, by known form the expressed as the constants of the deformation properties of material, geometric parameters of shell and effort/force T_{10}, T_{20}, S_0.

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For the determination of prolonged and instantaneous critical loads to obtained system (5.9.8) are used Hurwitz' criteria.

2. Let us examine stability of circular closed cylindrical

shell, by compressed along forming force T_{10}. At zero time, let us give to shell disturbance/perturbation in the form of low initial sagging and angle of rotation of the axisymmetric form

$$w=f\sin\frac{m\pi x}{l};\quad \gamma_x=\chi\cos\frac{m\pi x}{l} \tag{5.9.9}$$

and it is investigated the further behavior of sagging/deflection. Let us introduce the function of stresses F and satisfy, thus, to the first two equations of equilibrium (5.9.4):

$$\sigma_x h=T_1=\frac{\partial^2 F}{\partial y^2};\quad \sigma_y h=T_2=\frac{\partial^2 F}{\partial x^2}; \tag{5.9.10}$$

$$\sigma_{xy}h=S=\frac{\partial^2 F}{\partial x\partial y},$$

where σ_x, σ_y, τ_{xy} - the membrane stresses of shell.

Eliminating in three last/latter equations (5.9.4) Q_1, Q_2 and taking into account subsequently, that all values depend only on coordinate x, taking into account (5.9.5) we have

$$\frac{2h^3}{3}A_{1111}\frac{\partial^3\gamma_x}{\partial x^3}-\frac{1}{R}\frac{\partial^2 F}{\partial x^2}-\frac{\partial^2 w}{\partial x^2}T_{10}=0, \tag{5.9.11}$$

and from (5.9.4) taking into account (5.9.5) -

$$\frac{2h^3}{3} A_{1111} \frac{\partial^2 \gamma_x}{\partial x^2} = A_{1313} 2h \left(\gamma_x + \frac{\partial w}{\partial x} \right) -$$

$$-2h \int_0^t R_{1313}(t-\theta) \left(\gamma_x + \frac{\partial w}{\partial x} \right) d\theta. \qquad (5.9.12)$$

From the equation of the consistency of strains (5.9. 3) taking into account (5.9. 2), (5.9. 10) and

$$\varepsilon_{22} = a_{2222}\sigma_y + a_{2211}\sigma_x \qquad (5.9.13)$$

we have

$$\frac{1}{2h} a_{2222} \frac{\partial^4 F}{\partial x^4} - \frac{1}{R} \frac{\partial^2 w}{\partial x^2} = 0. \qquad (5.9.14)$$

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Set/assuming

$$R_{1313} = \frac{A_{1313} - B_{1313}}{n_{13}} e^{-\frac{t-\theta}{n_{13}}} \qquad (5.9.15)$$

and after eliminating from (5.9. 11), (5.9. 12) and (5.9. 14) F and γ_x, we obtain for f the following equation:

$$n_{13} a_1 \dot{f} + a_0 f = 0, \qquad (5.9.16)$$

where

$$a_1=A_{1313}2h\frac{m\pi}{l}+A_{1313}2h\frac{1}{D_{11}}\left[\frac{1}{R^2}\frac{2h}{a_{2222}}\left(\frac{l}{m\pi}\right)^3-\frac{l}{m\pi}T_{10}\right]+$$
$$+\left(\frac{1}{R^2}\frac{2h}{a_{2222}}\frac{l}{m\pi}-\frac{m\pi}{l}T_{10}\right);$$
$$a_0=B_{1313}2h\frac{m\pi}{l}+B_{1313}2h\frac{1}{D_{11}}\left[\frac{1}{R^2}\frac{2h}{a_{2222}}\left(\frac{l}{m\pi}\right)^3-\frac{l}{m\pi}T_{10}\right]+$$
$$+\left(\frac{1}{R^2}\frac{2h}{a_{2222}}\frac{l}{m\pi}-\frac{m\pi}{l}T_{10}\right);$$
$$D_{11}=\frac{2A_{1111}h^3}{3}.$$

Let us note that f and χ are connected linearly:

$$\chi=\frac{1}{D_{11}}\left[\frac{1}{R^2}\frac{2h}{a_{2222}}\left(\frac{l}{m\pi}\right)^3-\left(\frac{l}{m\pi}\right)T_{10}\right]f. \qquad (5.9.17)$$

The initial sagging/deflection w will not grow/rise in time, if are satisfied the conditions

$$a_1>0 \quad \text{and.} \quad a_0>0. \qquad (5.9.18a,b)$$

From (5.9.18a) we obtain instantaneous critical force taking into account the strains of the transverse shift/shears:

$$T_{10}{}^{\text{M}}=\frac{D_{11}\frac{\lambda_{\text{M}}^2}{R^2}}{1+\frac{D_{11}\frac{\lambda_{\text{M}}^2}{R^2}}{2hA_{1313}}}+\frac{2h}{a_{2222}\lambda_{\text{M}}^2}; \quad \lambda_{\text{M}}=\frac{m\pi R}{l}. \qquad (5.9.19)$$

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Minimizing (5.9.19) on λ_M, we have:

$$\lambda_M^2=\frac{\frac{4D_{11}}{a_{2222}A_{1313}R^2}\pm\frac{4}{R}\sqrt{\frac{2D_{11}h}{a_{2222}}}}{\frac{4D_{11}}{R^2}-\frac{8h}{a_{2222}}\left(\frac{D_{11}}{2hA_{1313}R^2}\right)^2}. \qquad (5.9.20)$$

From (5.9.18b) we obtain prolonged critical force taking into account the transverse shift/shears:

$$T_{10Д}=\frac{D_{11}\frac{\lambda_Д^2}{R^2}}{1+\frac{D_{11}\frac{\lambda_Д^2}{R^2}}{2hB_{1313}}}+\frac{2h}{a_{2222}\lambda_Д^2}; \qquad (5.9.21)$$

$$\lambda_Д^2=\frac{\frac{4D_{11}}{a_{2222}B_{1313}R^2}\pm\frac{4}{R}\sqrt{\frac{2D_{11}h}{a_{2222}}}}{\frac{4D_{11}}{R^2}-\frac{8h}{a_{2222}}\left(\frac{D_{11}}{2hB_{1313}R^2}\right)^2}. \qquad (5.9.22)$$

For the case - the stability of cylindrical shell - the process of bulge in time and prolonged critical load in question cannot be investigated with the aid of kirkhgofa - Love's model, according to which we here obtain only instantaneous critical force $T_{10}{}^{\mathfrak{s}}$. The process of loss of stability in time in this case can be investigated taking into account the strains of transverse shift/shears. Assumption about the absence of transverse shift/shears (kirkhgofa - Love's model) corresponds to the acceptance of infinite shear moduli:

$$A_{1313}=B_{1313}=\infty,$$

that, according to (5.9.19) and (5.9.21), it is led to the instantaneous critical force

$$T_{10}{}^{\mathfrak{s}}=D_{11}\frac{\lambda^2}{R^2}+\frac{2h}{a_{2222}\lambda^2}, \qquad (5.9.23)$$

where after minimization for λ^2 we have

$$\lambda^2=\sqrt{\frac{R^2 2h}{D_{11}a_{2222}}}. \tag{5.9.24}$$

3. For illustration let us take numerical example. Let us examine the cylindrical shell, manufactured from the glass-fiber-reinforced plastic, reinforced along generatrix and in transverse direction, with the following characteristics:

$$E_1=A_{1111}=200\,000\ \text{кг/см}^2{}^{(1)};\quad E_2=\frac{1}{a_{2222}}=100\,000\ \text{кг/см}^2{}^{(1)};$$
$$G_{13}=A_{1313}=5000\ \text{кг/см}^2{}^{(1)};\quad G_{13}{}^{\text{Д}}=B_{1313}=1500\ \text{кг/см}^2{}^{(1)}.$$

Key: (1). kg/m².

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Geometric characteristics of the shell: R = 10 cm, h = 0.1 cm.

If let us accept kirkhgofa - Love's hypothesis, then we will obtain only instantaneous critical force according to (5.9.23) and (5.9.24): $T_{10}{}^{\text{э}}=326$ kg/cm; λ^2 = 122.

Taking into account transverse shift/shears, we obtain instantaneous (5.9.19) and prolonged (5.9.21) the critical forces: $T_{10}{}^{\text{M}}=300$ kg/cm and $\lambda_{\text{M}}{}^2=139$; $T_{10}{}^{\text{Д}}=237$ kg/cm and $\lambda_{\text{Д}}{}^2=268$.

Let us estimate the errors, permissible as a result of the disregard of the transverse shift/shears:

$$\frac{T_{10}{}^{\text{Э}} - T_{10}{}^{\text{M}}}{T_{10}{}^{\text{M}}} 100 \approx 9\%; \quad \frac{T_{10}{}^{\text{Э}} - T_{10}{}^{\text{Д}}}{T_{10}{}^{\text{Д}}} 100 \approx 38\%.$$

Thus, the disregard of the phenomena, connected with transverse shift/shear, in certain cases is led to essential errors.

§5.10. Stability of orthotropic slanting spherical shells with large sagging/deflections taking into account the strains of transverse shift/shears and creep.

We investigate stability during creep of slanting axisymmetric shell with a radius of a, loaded by the axisymmetric transverse load p. In this case, is considered only interlayer shift creep, and also creep in the plane of layers of the reinforcing fabric. Let us examine the sagging/deflections, commensurable with the thickness of shell, and strain component let us accept lcw. Nondeformed median surface of shell is accepted as so slanting that it is described by the nonlinear relationship/ratios of circular plates for the bent state, s.uceton, however, that that in the initial unloaded state in

the shell of stress and strain they are absent [298]. Let us compose the initial equations of task taking into account the adopted assumptions.

1. Equations of equilibrium of flexible circular plate take following form [64, 298]:

from projections of forces in radial direction

$$r\frac{\partial N_r}{\partial r}+N_r-N_\varphi=0, \qquad (5.10.1a)$$

also in vertical direction

$$Q_r+N_r\frac{\partial W}{\partial r}+\frac{1}{r}\int_0^r prdr=0. \qquad (5.10.1b)$$

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Equation of moment balance

$$\frac{\partial M_r}{\partial r}+\frac{1}{r}(M_r-M_\varphi)=-Q_r. \qquad (5.10.1c)$$

Let us pass to slanting, circular cross-section shell and describe its deformed surface by the expression of the warped surface of flexible circular plate (Fig. 5.1) [298]:

$$W=-(y-w)=w-y. \qquad (5.10.2)$$

Here y is an ordinate of nondeformed median surface of shell; w - the sagging/deflection of shell.

Equations (5.10.1a) and (5.10.1c) will remain previous, and instead of (5.10.1b) we have

$$-Q_r+N_r\frac{\partial(w-y)}{\partial r}+\frac{1}{r}\int_0^r prdr=0. \qquad (5.10.1\,d)$$

2. Geometric relationship/ratios. Let us examine two versions of the account of transverse shift/shears.

a) let us assume that the filament, normal to nondeformed median surface, in the process of the deformation of shell only is turned, without experience/testing elongation - compression. For this case we have the following geometric constraints between strains on equidistant and median surfaces [229]:

$$e_r=\varepsilon_r+z\varkappa_r;\quad e_\varphi=\varepsilon_\varphi+z\varkappa_\varphi;\quad e_{rz}=\varepsilon_{rz}, \qquad (5.10.3\,a,b,c)$$

here e_{ij} are strains of equidistant surface;

ε_{ij} - the strain of median surface, moreover

$$\varepsilon_r=\frac{\partial u}{\partial r}+\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2-\frac{\partial w}{\partial r}\frac{\partial y}{\partial r};$$

$$\varepsilon_\varphi=\frac{u}{r};$$

$$\varepsilon_{rz}=\frac{\partial w}{\partial r}-\gamma_r;\quad \varkappa_r=-\frac{\partial \gamma_r}{\partial r};\quad \varkappa_\varphi=-\frac{\partial \gamma_\varphi}{\partial \varphi},\ (5.10.4\,a,b,c,d,e)$$

where γ_r **is an angle of rotation of standard.**

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After eliminating from (5.10.4a, b) displacement u, we will obtain the equation of the consistency:

$$\varepsilon_r-\varepsilon_\varphi-r\frac{\partial \varepsilon_\varphi}{\partial r}=\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2-\frac{\partial w}{\partial r}\frac{\partial y}{\partial r}. \qquad (5.10.5)$$

b) taking into account also the bending of the normal filaments of strain on equivalent surface will be (expressions of the strains of median surface will remain previous [230]):

$$e_r=\varepsilon_r+z\frac{\partial \gamma_r}{\partial r}-\frac{z^3}{3h^2}\left(\frac{\partial^2 w}{\partial r^2}-\frac{\partial \gamma_r}{\partial r}\right);$$

$$e_\varphi=\varepsilon_\varphi=\frac{u}{r}; \qquad (5.10.6\,a,b,c)$$

$$e_{rz}=\varepsilon_{rz}\left(1-\frac{z^2}{h^2}\right)=\left(\frac{\partial w}{\partial r}-\gamma_r\right)\left(1-\frac{z^2}{h^2}\right).$$

3. Physical relationship/ratios. As in the preceding/previous paragraphs, it is considered only shift creep of interlayer shift/shear and shift/shear in the plane of layers of the reinforcing fabric, after considering that in the fiber direction creep from normal stresses is absent:

$$\begin{aligned}
&\sigma_r=A_{11}e_r+A_{12}e_\varphi;\\
&\sigma_\varphi=A_{22}e_\varphi+A_{21}e_r;\\
&\tau_{r\varphi}=G_{33}e_{r\varphi}-\int_0^t R_{r\varphi}(t-\theta)\,e_{r\varphi}(\theta)\,d\theta; \qquad (5.10.7\,a,b,c,d,e)\\
&\tau_{rz}=G_{44}e_{rz}-\int_0^t R_{rz}(t-\theta)\,e_{rz}(\theta)\,d\theta;\\
&\tau_{\varphi z}=G_{55}e_{\varphi z}-\int_0^t R_{\varphi z}(t-\theta)\,e_{\varphi z}(\theta)\,d\theta.
\end{aligned}$$

Taking into account (5.10.3) the expression of internal effort/forces, is taken the form

$$\begin{aligned}
&M_r=\int_{-h}^{h}\sigma_r z\,dz=-\frac{2h^3}{3}\left(A_{11}\frac{\partial\gamma_r}{\partial r}+A_{12}\frac{\gamma_r}{r}\right);\\
&M_\varphi=\int_{-h}^{h}\sigma_\varphi z\,dz=-\frac{2h^3}{3}\left(A_{22}\frac{\gamma_r}{r}+A_{12}\frac{\partial\gamma_r}{\partial r}\right); \qquad (5.10.8\,a,b,c)
\end{aligned}$$

$$Q_r=\int\limits_{-h}^{h}\tau_{rz}dz=2hG_{rz}\left(\frac{\partial w}{\partial r}-\gamma_r\right)-$$

$$-2h\int\limits_0^t R_{12}(t-\theta)\left(\frac{\partial w}{\partial r}-\gamma_r\right)d\theta.$$

Page 293. Upon consideration of strains, according to (5.10.6), we have the following expressions of internal effort/forces:

$$M_r=-\frac{2h^3}{3}\left\{\frac{4}{5}\left(A_{11}\frac{\partial\gamma_r}{\partial r}+A_{12}\frac{\gamma_r}{r}\right)-\frac{1}{5}\left(A_{11}\frac{\partial^2 w}{\partial r^2}-\right.\right.$$

$$\left.\left.-A_{12}\frac{1}{r}\frac{\partial w}{\partial r}\right)\right\};$$

$$M_\varphi=-\frac{2h^3}{3}\left\{\frac{4}{5}\left(A_{22}\frac{\gamma_r}{r}+A_{12}\frac{\partial\gamma_r}{\partial r}\right)-\frac{1}{5}\left(A_{22}\frac{1}{r}\frac{\partial w}{\partial r}+\right.\right.$$

$$\left.\left.+A_{12}\frac{\partial^2 w}{\partial r^2}\right)\right\};$$

$$Q_r=\frac{4}{3}hG_{rz}\left(\frac{\partial w}{\partial r}-\gamma_r\right)-\frac{4}{3}\int\limits_0^t R_{rz}(t-\theta)\left(\frac{\partial w}{\partial r}-\gamma_r\right)d\theta.$$

(5.10.9 *a*, *b*, *c*)

4. Resolving equations of task we will obtain, utilizing in equations of equilibrium and consistency physical relationship/ratios. We determine normal effort/forces by the function of stresses F in the form

$$N_r = \frac{F}{r}; \quad N_\varphi = \frac{\partial F}{\partial r} \qquad (5.10.10\ a, b)$$

and by this we satisfy the equation of equilibrium (5.10.1a). Equations (5.10.1b) and (5.10.1c) let us present in the form:

$$-Q_r + N_r \frac{\partial(w-y)}{\partial r} + \frac{1}{r}\int_0^r prdr = 0;$$

$$(5.10.11\ a, b)$$

$$\frac{\partial M_r}{\partial r} + \frac{1}{r}(M_r - M_\varphi) + N_r \frac{\partial(w-y)}{\partial r} + \frac{1}{r}\int_0^r prdr = 0.$$

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a) during the use in (5.10.11a, b) of physical relationship/ratios, according to (5.10.8), when is considered only the rotation of standards, we have:

$$2hG_{rz}\left(\gamma_r-\frac{\partial w}{\partial r}\right)-\frac{\partial(w-y)}{\partial r}\frac{F}{r}-2h\int_0^t R_{rz}(t-\theta)\left(\gamma_r-\right.$$
$$\left.-\frac{\partial w}{\partial r}\right)d\theta-\frac{1}{r}\int_0^r prdr=0;$$
$$\frac{2h^3}{3}\left[A_{11}\left(\frac{\partial^2\gamma_r}{\partial r^2}+\frac{1}{r}\frac{\partial\gamma_r}{\partial r}\right)-A_{22}\frac{\gamma_r}{r^2}\right]-\frac{F}{r}\frac{\partial(w-y)}{\partial r}+$$
$$+\frac{1}{r}\int_0^r prdr=0. \qquad (5.10.12\,a,b)$$

Further in the equation of consistency (5.10.5) we utilize physical relationship/ratios (5.10.7), solved relative to the components of tensor of the strains:

$$\varepsilon_r=\frac{1}{2h}(a_rN_r+a''N_\varphi);$$
$$\varepsilon_\varphi=\frac{1}{2h}(a_\varphi N_\varphi+a''N_r). \qquad (5.10.13\,a,b)$$

that it leads to the expression

$$\frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2-\frac{\partial w}{\partial r}\frac{\partial y}{\partial r}=\frac{1}{2h}\left[a_r\frac{F}{r}-a_\varphi\left(\frac{\partial F}{\partial r}+r\frac{\partial^2F}{\partial r^2}\right)\right]. \qquad (5.10.14)$$

The obtained expressions (5.10.12a), (5.10.12b) and (5.10.14) they compose the complete system of equations of task of relatively

three independent variables: w, γ and F.

b) the resolving equations with the use of geometric relationship/ratios (5.10.6a, b, c) we will obtain during substitution/replacement (5.10.9a)-(5.10.9c) in (5.10.11a, b); the equation of consistency (5.10.14) it remains previous:

$$\frac{4}{3}hG_{rz}\left(\gamma_r-\frac{\partial w}{\partial r}\right)-\frac{4}{3}h\int_0^t R_{rz}(t-\theta)\left(\gamma_r-\frac{\partial w}{\partial r}\right)d\theta-$$

$$-\frac{F}{r}\frac{\partial(w-y)}{\partial r}-\frac{1}{r}\int_0^r prdr=0;$$

$$\frac{2h^3}{3}\left\{\frac{4}{5}\left[A_{11}\frac{\partial^2\gamma_r}{\partial r^2}+\frac{1}{r}A_{12}\frac{\partial\gamma_r}{\partial r}-\frac{1}{r^2}\left(A_{22}-A_{12}\right)\gamma_r\right]+\right.$$

$$\left.+\frac{1}{5}\left[A_{11}\frac{\partial^3 w}{\partial r^3}+\frac{1}{r}A_{12}\frac{\partial^2 w}{\partial r^2}-\frac{1}{r^2}\left(A_{22}-A_{12}\right)\frac{\partial w}{\partial r}\right]\right\}- \qquad (5.10.15\,a,b)$$

$$-\frac{F}{r}\frac{\partial(w-y)}{\partial r}-\frac{1}{r}\int_0^r prdr=0.$$

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c) let us note that, utilizing in this task kirkhgofa - Love's model, we obtain only elastic solution, since here is not considered

interlayer shift creep. Utilizing in the equation of equilibrium (5.10.11b) of the expression of internal torque, calculated under the assumption of the nondeformability of the standards:

$$M_r = -\frac{2h^3}{3}\left(A_{11}\frac{\partial^2 w}{\partial r^2} + A_{12}\frac{1}{r}\frac{\partial w}{\partial r}\right);$$
$$M_\varphi = -\frac{2h^3}{3}\left(A_{22}\frac{1}{r}\frac{\partial w}{\partial r} + A_{21}\frac{\partial^2 w}{\partial r^2}\right). \qquad (5.10.16a, b)$$

we obtain together with (5.10.14) two resolving equations of the task:

$$\frac{2h^3}{3}\frac{d}{dr}\left(A_{11}\frac{\partial^2}{\partial r^2} + A_{12}\frac{1}{r}\frac{\partial}{\partial r}\right)w + \frac{1}{r}\frac{2h^3}{3}\left[(A_{11}-A_{21})\frac{\partial^2}{\partial r^2} - \right.$$
$$\left. -(A_{22}-A_{12})\frac{1}{r}\frac{\partial}{\partial r}\right]w - \frac{F}{r}\frac{\partial(w-y)}{\partial r} - \frac{1}{r}\int_0^r pr\,dr = 0. \qquad (5.10.17)$$

In this case, after assuming the infinite interlayer shift rigidity of material, we obtain the high values of critical loads.

5. Mode/conditions particular task of stability of slanting spherical shell with constant, evenly distributed load p. For this case we have (Fig. 5.2)

$$\frac{1}{r}\frac{\partial y}{\partial r} = -\frac{1}{R}. \qquad (5.10.18)$$

We utilize kinematic relationship/ratios (5.10.3) and (5.10.4). The inertial forces let us disregard, since this is led to the distorted results only in the zone of cracking where the rate of the increase of sagging/deflections has considerably more preceding/previous stages [233].

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We approximate dimensionless quantities $\frac{F}{A_{11}a^2}$, $\frac{w}{a}$, and γ_r by the following polynomials:

$$\frac{F}{A_{11}a^2}=b_0+b_1\rho+b_2\rho^2;$$

$$\frac{w}{a}=C_0+C_1\rho+C_2\rho^2;$$

$$\gamma_r=g_0+g_1\rho+g_2\rho^2. \qquad (5.10.19\,a,b,c)$$

where $\rho=\frac{r}{a}$.

The constants of polynomials (5.10.19) let us determine from boundary conditions. In the examination of hinged support, we have:

$$u=w=M_r=0 \text{ when } r=a; \qquad (5.10.20a)$$

$$N_r=N_\varphi, \quad \gamma_r=0, \quad \frac{\partial w}{\partial r}=0 \text{ with. } r=0. \qquad (5.10.20b)$$

From condition $N_r=N_\varphi$ with r = 0, we have $\frac{F(r=0)}{A_{11}a^2}=b_0=0$. From $u|_{r=a}=0$ we have $\frac{u(r=a)}{r}=e_\varphi(r=a)=\frac{1}{2h}\left[a_\varphi\frac{\partial F(r=a)}{\partial r}+a''\frac{F(r=a)}{r}\right]=0,$

whence $b_1=-b_2\frac{2a_\varphi+a''}{a_\varphi+a''}$.

If we designate $b_2 = b(t)$, we will obtain

$$F=A_{11}a^2(\rho-\tilde{a})\rho b(t), \qquad (5.10.21)$$

where $\tilde{a}=\frac{2a_\varphi+a''}{a_\varphi+a''}$.

From condition $\frac{\partial w(r=0)}{\partial r}=0$ we have $C_1 = 0$. From $\frac{w(r=a)}{a}=0$ we obtain $C_0 = -C_2$.

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After designating $C_0 = c(t)$, we come to

$$w=a(1-\rho^2)c(t). \qquad (5.10.22)$$

From $\gamma(r = 0) = 0$ we have $g_0 = 0$, while from

$$M_r(r=a)=\frac{2h^3}{3}\left[A_{11}\frac{\partial\gamma(r=a)}{\partial r}+A_{12}\frac{\gamma(r=a)}{r}\right]=0$$

we obtain $g_1 = g_2\tilde{A}$, where $\tilde{A}=\frac{2A_{11}+A_{12}}{A_{11}+A_{12}}$.

If we designate $g_2 = g(t)$, we come to

$$\gamma_r=\rho(\rho-\tilde{A})g(t). \qquad (5.10.23)$$

For the determination of the functions of time b(t), c(t) and g(t) we apply to equations (5.10.12a), (5.10.12b), (5.10.14) bubnov - Galerkin's method and is passed to dimensionless variable $\rho = r/a$:

$$\int_0^1(1-\rho^2)\left\{2hG\left(\gamma_r-\frac{1}{a}\frac{\partial w}{\partial\rho}\right)+\frac{F}{a\rho}\left(\frac{1}{a}\frac{\partial w}{\partial\rho}+a\rho\frac{1}{R}\right)-\right.$$

$$\left.-2h\int_0^t\tilde{G}(t-\theta)\left(\gamma_r-\frac{1}{a}\frac{\partial w}{\partial\rho}\right)d\theta+\frac{pa\rho}{2}\right\}d\rho=0;$$

$$\int_0^1\rho(\rho-\tilde{A})\left\{\frac{2h^3}{3}\left[A_{11}\left(\frac{1}{a^2}\frac{\partial^2\gamma_r}{\partial\rho^2}+\frac{1}{a^2\rho}\frac{\partial\gamma_r}{\partial\rho}\right)-A_{22}\frac{\gamma_r}{a^2\rho^2}\right]-\right.$$

$$\left.-\left[\frac{f}{a\rho}\left(\frac{1}{a}\frac{\partial w}{\partial\rho}+a\frac{\rho}{R}\right)+\frac{pa\rho}{2}\right]\right\}d\rho=0; \quad (5.10.24\,a,b,c)$$

$$\int_0^1\rho(\rho-\tilde{a})\left\{\frac{1}{2a^2}\left(\frac{\partial w}{\partial\rho}\right)^2+\frac{\rho}{R}\frac{\partial w}{\partial\rho}+\frac{1}{2h}\left[a_\varphi\left(\frac{1}{a}\frac{\partial F}{\partial\rho}+\frac{\rho}{a}\frac{\partial^2F}{\partial\rho^2}\right)-\right.\right.$$

$$\left.\left.-a_r\frac{F}{a\rho}\right]\right\}d\rho=0.$$

Subsequently let us examine shell of the monotropic material:

$$\varepsilon_r = \frac{1}{E}(\sigma_r - \nu\sigma_\varphi); \quad \sigma_r = \frac{E}{1-\gamma^2}(\varepsilon_r + \nu\varepsilon_\varphi);$$

$$\varepsilon_\varphi = \frac{1}{E}(\sigma_\varphi - \nu\sigma_r); \quad \sigma_\varphi = \frac{E}{1-\nu^2}(\varepsilon_\varphi + \nu\varepsilon_r); \quad (5.10.25\,a,b,c,d,e)$$

$$\tau_{rz} = G_{rz}\gamma_{rz} - \int_0^t \tilde{G}(t-\theta)\gamma_{rz}(\theta)\,d\theta.$$

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Experimental data are described well by nucleus in the form, proposed by Yu. N. Rabotnov [156, 197]:

$$\tilde{G}(t-\theta) = G^*(t-\theta)^\alpha \sum_{k=0}^{\infty} \frac{\varkappa^k (t-\theta)^{k(1+\alpha)}}{\Gamma[(k+1)(1+\alpha)]}. \quad (5.10.26)$$

After integration from system (5.10.24a, b, c) taking into account (5.10.21)-(5.10.23) we have:

$$G\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)g(t)+\frac{1}{2}Gc(t)-\frac{Ea}{2h}\left(\frac{\tilde{a}}{4}-\frac{2}{15}\right)\frac{a}{R}b(t)+$$
$$+\frac{Ea}{h}\left(\frac{\tilde{a}}{4}-\frac{2}{15}\right)c(t)b(t)+\frac{pa}{16h}-\int_0^t\tilde{G}(t-\theta)\times$$
$$\times\left[\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)g(\theta)+\frac{1}{2}c(\theta)\right]d\theta=0;$$

$$\frac{2h^3}{a^3}\left(\frac{\tilde{A}}{2}-\frac{1}{3}\right)g(t)-\frac{p}{2E}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)+\frac{a}{R}\left[\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)\frac{\tilde{\tilde{a}}}{2}-\right.$$
$$\left.-\left(\frac{\tilde{A}}{4}-\frac{1}{5}\right)\right]b(t)-2\left[\tilde{a}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)-\left(\frac{\tilde{A}}{4}-\frac{1}{5}\right)\right]c(t)b(t)=0;$$

$$b(t)=\frac{4}{3}\frac{h}{R}\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}c(t)-\frac{4}{3}\frac{h}{a}\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}c(t)^2. \quad (5.10.27\,a,b,c)$$

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After solving system (5.10.27) relative to c(t) with t = 0, we will obtain equation for the solution of the instantaneous-elastic problem, necessary for the initial conditions:

$$Bc(0)^3-Dc(0)^2+Hc(0)=A\overline{p}, \qquad (5.10.28)$$

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where

$$A=\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)\frac{\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)a^{3}}{\left(\frac{A}{2}-\frac{1}{3}\right)4h^{3}}-\frac{aE}{16hG};$$

$$B=\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)\left[\tilde{a}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)\left(\frac{\grave{A}}{4}-\frac{1}{5}\right)\right]\frac{4}{3}\left(\frac{a}{h}\right)^{2}\times$$

$$\times\frac{\frac{\grave{a}}{4}-\frac{1}{5}}{\left(\frac{\tilde{a}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{2}-\frac{1}{3}\right)}-\left(\frac{a}{4}-\frac{2}{15}\right)\frac{4}{3}\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}\frac{E}{G};$$

$$D=\frac{\tilde{A}}{4}-\frac{4}{15R}\left[\tilde{a}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{4}-\frac{1}{5}\right)\right]\frac{a^{2}}{h^{2}}\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\left(\frac{\tilde{a}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{2}-\frac{1}{3}\right)}-$$

$$-2\frac{a}{R}\left(\frac{\tilde{a}}{4}-\frac{2}{15}\right)\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}\frac{E}{G};$$

$$H=\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)\frac{a}{R}\left[\tilde{a}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{4}-\frac{1}{5}\right)\right]\frac{2}{3}\frac{h}{R}\left(\frac{a}{h}\right)^{3}\times$$

$$\times\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\left(\frac{\tilde{a}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{2}-\frac{1}{3}\right)}-\left[\left(\frac{\tilde{a}}{4}-\frac{2}{15}\right)\left(\frac{a}{R}\right)^{2}\frac{2}{3}\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}\frac{E}{G}-\frac{1}{2}\right];$$

$$\bar{p}=\frac{p}{E}.$$

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In the specific case of nucleus (5.10.26) with α = 0, after assuming $\varkappa=-\frac{1}{n}$ and $G^{*}=\frac{G-G^{Д}}{n}$ (exponential nucleus) from system (5.10.27), after solving its relatively c(t) and after passing to dimensionless time τ = t/n, we obtain the nonlinear differential equation

$$B'c(\tau)^3-D'c(\tau)^2+H'c(\tau)+3Bc(\tau)^2\dot{c}(\tau)-2Dc(\tau)\dot{c}(\tau)+ +H\dot{c}(\tau)=A'\bar{p}, \qquad (5.10.29)$$

where

$$A'=\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)\frac{\frac{\tilde{A}}{3}-\frac{1}{4}}{\frac{\tilde{A}}{2}-\frac{1}{3}}\frac{a^3}{4h^3}-\frac{a}{16h}\frac{E}{G^{Д}};$$

$$B'=\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)\left[\tilde{a}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{4}-\frac{1}{5}\right)\right]\frac{4}{3}\left(\frac{a}{h}\right)^2\times$$

$$\times\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\left(\frac{\tilde{a}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{2}-\frac{1}{3}\right)}-\left(\frac{\tilde{a}}{4}-\frac{2}{15}\right)\frac{4}{3}\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}\frac{E}{G^{Д}};$$

$$D'=\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)\left[\tilde{a}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)-\left(\frac{\tilde{A}}{4}-\frac{1}{5}\right)\right]\frac{2h}{R}\frac{a^3}{h^3}\times$$

$$\times\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\left(\frac{\tilde{a}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{2}-\frac{1}{3}\right)}-2\frac{a}{R}\left(\frac{\tilde{a}}{4}-\frac{2}{15}\right)\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}-\frac{E}{G^{Д}};$$

$$H'=\left(\frac{\tilde{A}}{4}-\frac{2}{15}\right)\frac{a}{R}\left[\tilde{a}\left(\frac{\tilde{A}}{3}-\frac{1}{4}\right)-\left(\frac{\tilde{A}}{4}-\frac{1}{5}\right)\right]\frac{2}{3}\frac{h}{R}\frac{a^3}{h^3}\times$$

$$\times \frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\left(\frac{\tilde{a}}{3}-\frac{1}{4}\right)\left(\frac{\tilde{A}}{2}-\frac{1}{3}\right)}-\left[\left(\frac{\tilde{a}}{4}-\frac{2}{15}\right)\left(\frac{a}{R}\right)^2\frac{2}{3}\frac{\frac{\tilde{a}}{4}-\frac{1}{5}}{\frac{\tilde{a}}{3}-\frac{1}{4}}\frac{E}{G_Д}-\frac{1}{2}\right].$$

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For the case when with $v = -\dot{c}(t) = 0$, we obtain dependence for determining the sagging/deflections of shell with the long effective load:

$$A'\bar{p} = B'c^3 - D'c^2 + H'c. \tag{5.10.30}$$

6. Numerical solution of particular task. Let us accept the following geometric parameters of shell (Fig. 5.2): $S/a = 0.07$; $a/h = 140$; $R/h = 1005$; $a/R = 0.139$; $\beta = 8°00'$; we take also the mechanical characteristics of monotropic material (glass-fiber-reinforced plastic): $E = 200{,}000$ kgf/cm²; $E/G = 30$; $\frac{E}{G_Д} = 150$; $\gamma = 0.1$. The coefficients of equations (5.10.28) - (5.10.30)

with the taken values of geometric parameters and the characteristics of material acquire the following values:

$$\begin{array}{ll} A=87\,238; & A'=86\,190; \\ B=3293{,}7; & B'=3248{,}5; \\ D=519{,}64; & D'=510{,}2; \\ H=8{,}29; & H'=7{,}82. \end{array} \qquad (5.10.31)$$

Instantaneous-elastic sagging/deflections and sagging/deflections in the case of the long effective load when $\tau = \infty$ are obtained according to expressions (5.10.28) and (5.10.30). The obtained peak loads can be call/named instantaneous and prolonged critical loads. In this example prolonged the critical load on 11o/o lower than instantaneous. Equation (5.10.29) at the values of coefficients (5.10.31) was solved on BESM-2.

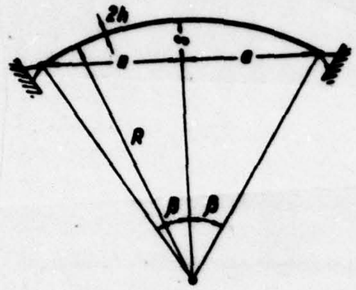

Fig. 5.2. Slanting spherical shell and its basic geometric parameters.

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Were undertaken the following values of the dimensionless parameter of the load: $1.71 \cdot 10^{-7}$; $2.34 \cdot 10^{-7}$; $2.71 \cdot 10^{-7}$; $2.78 \cdot 10^{-7}$; $2.80 \cdot 10^{-7}$; $2.92 \cdot 10^{-7}$; $3.00 \cdot 10^{-7}$. Solutions (5.10.29) at the taken values of $\bar{p}$ in the form of graphs are given in Fig. 5.3. From these graphs it follows that with the loads smaller than the prolonged critical, sagging/deflections in time are stabilized. When the load $\bar{p}^{д}<\bar{p}<\bar{p}^{м}$ the sagging/deflections grow at the increasing velocity; in this case from the obtained graphs, it is possible to calculate the critical time which is determined by conditional point on graph c ~ τ.

§5.11. Stability of cylindrical shell during the nonlinear creep of the material.

The broad class of polymeric materials follows the nonlinear laws of creep [154]. For the analysis of stability of cylindrical shell, let us accept communication/connection between deformations, stresses and their velocities in the form of nonlinear-differential dependence. To this corresponds the following differential expression of the function of the local deformations:

$$\gamma_{zx}=\frac{\sigma_{zx}}{G}+\frac{n\dot{\sigma}_{zx}}{G}+Q\varepsilon_{zx}{}^{\alpha}-nB\dot{\varepsilon}_{zx}, \qquad (5.11.1)$$

where n, G, B, Q and α - constant of material. We take, that the function of local deformations has identical value both with the loading and during unloading of material (for some polymeric materials this is confirmed by experiments).

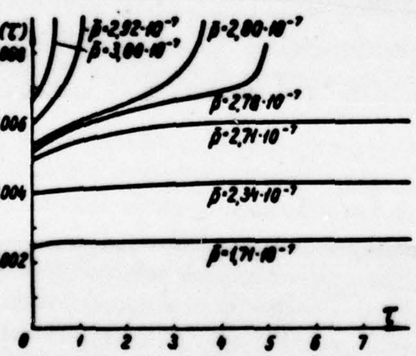

Fig. 5.3. Diagrams c-τ with transverse load $\bar{p}$ of different intensity (results are calculated with the aid of BESM-2).

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We will consider inertialess subcritical state of the shell, compressed along generatrices, i.e., $\sigma_{ij}=0$, besides $\sigma_{11}\neq 0$. After examining low deviations from the ground state, the disturbed state let us describe by the varied law, utilizing a recording (5.2.7):

$$\delta\varepsilon_{ij}=\frac{1}{S}\int_S(\delta\gamma_{xz}v_{ij}+\gamma_{xz}\delta v_{ij})\,ds. \qquad (5.11.2)$$

Taking into account (5.11.1) for a planar-stressed state we have:

$$\delta\varepsilon_{11}=\frac{2}{15G}\delta\sigma_{11}-\frac{1}{15G}\delta\sigma_{22}+\frac{2n}{15G}\delta\dot{\sigma}_{11}-\frac{n}{15G}\delta\dot{\sigma}_{22}+\frac{8}{105}Q\varepsilon_{11}^{2}\delta\varepsilon_{11}-$$

$$-\frac{4}{105}Q\varepsilon_{11}^{2}\delta\varepsilon_{22}-\frac{2n}{15}B\delta\dot{\varepsilon}_{11}+\frac{n}{15}B\delta\dot{\varepsilon}_{22};$$

$$\delta\varepsilon_{22}=\frac{1}{20G}\delta\sigma_{22}-\frac{1}{15G}\delta\sigma_{11}+\frac{n}{20G}\delta\dot{\sigma}_{22}-\frac{n}{15G}\delta\dot{\sigma}_{11}+$$

$$+\frac{1}{35}Q\varepsilon_{11}^{2}\delta\varepsilon_{22}-\frac{4}{105}Q\varepsilon_{11}^{2}\delta\varepsilon_{11}+\frac{n}{20}B\delta\dot{\varepsilon}_{22}+\frac{n}{15}B\delta\dot{\varepsilon}_{11}; \qquad (5.11.3)$$

$$2\delta\varepsilon_{12}=\frac{\pi}{8G}\delta\sigma_{12}+\frac{\pi n}{8G}\delta\dot{\sigma}_{12}+\frac{4}{105}Q\varepsilon_{11}^{2}\delta\varepsilon_{12}-\frac{nB}{5}\delta\dot{\varepsilon}_{12}.$$

We will use the obtained dependences for determining an increase in internal torque M_1, M_2, and H and in the increase in the deformations in median surface of shell. Let us accept further Kirchhoff-Love's hypothesis:

$$\delta\varepsilon_{11} = -zw_{xx}; \quad \delta\varepsilon_{22} = -zw_{yy}; \quad \delta\gamma = -2zw_{xy}, \quad (5.11.4)$$

let us introduce the function of the stresses:

$$\sigma_{11}h = N_1 = \frac{\partial^2\varphi}{\partial y^2}; \quad \sigma_{22}h = N_2 = \frac{\partial^2\varphi}{\partial x^2}; \quad \tau_{xy}h = S = -\frac{\partial^2\varphi}{\partial x\partial y} \quad (5.11.5)$$

we utilize equations of equilibrium and coincidence of the deformations:

$$\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 H}{\partial x\partial y} + \frac{\partial^2 M_2}{\partial y^2} - N_{10}\frac{\partial^2 w}{\partial x^2} + \frac{N_2}{R} = 0;$$
$$\frac{\partial^2\varepsilon_1}{\partial y^2} + \frac{\partial^2\varepsilon_2}{\partial x^2} - \frac{\partial^2\gamma}{\partial x\partial y} + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0. \quad (5.11.6)$$

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Using the physical relationship/ratios (5.11.3) in the equations of equilibrium and consistency of deformations (5.11.6) and after assuming the axisymmetric form of the bulge

$$w = f\sin\frac{m\pi x}{l}, \quad (5.11.7)$$

we obtain following system relative to the function of stresses ϕ and of sagging/deflections w:

$$n^2\ddot{\varphi}_{xxxx}+n\dot{\varphi}_{xxxx}=30RG\left(\frac{3}{4}-\frac{2}{105}Q\varepsilon_{11}{}^2\right)\frac{h^3}{12}n\dot{w}_{xxxxxx}+$$
$$+RGB\frac{h^3}{12}n^2\ddot{w}_{xxxxxx}+RN_{10}n\dot{w}_{xxxx}+$$
$$+RN_{10}n^2\ddot{w}_{xxxx};\tag{5.11.8}$$
$$n^2\ddot{\varphi}_{xxxx}+\frac{30}{B}\left[\left(\frac{3}{4}-\frac{2}{105}Q\varepsilon_{11}{}^2\right)+\frac{B}{30}\right]n\dot{\varphi}_{xxxx}+$$
$$+\frac{30}{B}\left(\frac{3}{4}-\frac{2}{105}Q\varepsilon_{11}{}^2\right)\varphi_{xxxx}=\frac{900G}{RB}\left[\left(\frac{3}{4}-\frac{2}{105}Q\varepsilon_{11}{}^2\right)\times\right.$$
$$\left.\times\left(\frac{2}{105}Q\varepsilon_{11}{}^2-2\right)+1\right]w_{xx}+$$
$$+\frac{900G}{RB}\left[\frac{B}{30}\left(\frac{4}{105}Q\varepsilon_{11}{}^2-\frac{11}{4}\right)\right]n\dot{w}_{xx}-\frac{GB}{R}n^2\ddot{w}_{xx}.$$

After eliminating from the obtained equations the function of stresses, relative to the amplitude of sagging/deflections f we have

$$n^2a_2\ddot{f}+na_1\dot{f}+a_0f=0,$$

where

$$a_2=BG\frac{h}{R}+RGB\left(\frac{m\pi}{l}\right)^4\frac{h^3}{12}-R\left(\frac{m\pi}{l}\right)^2N_{10};\tag{5.11.9}$$
$$a_1=60RG\left(\frac{m\pi}{l}\right)^4\left(\frac{3}{4}\frac{2}{105}Q\varepsilon_{11}{}^2\right)\frac{h^2}{12}+30G\frac{h}{R}\left(\frac{11}{4}-\right.$$
$$\left.-\frac{4}{105}Q\varepsilon_{11}{}^2\right)-\left[\frac{30}{B}\left(\frac{3}{4}-\frac{2}{105}Q\varepsilon_{11}{}^2\right)+1\right]\left(\frac{m\pi}{l}\right)^2RN_{10};$$
$$a_0=900\frac{G}{B}R\left(\frac{m\pi}{l}\right)^4\left(\frac{3}{4}-\frac{2}{105}Q\varepsilon_{11}{}^2\right)\frac{h^3}{12}-\frac{900Gh}{BR}\left[\left(\frac{3}{4}-\right.\right.$$
$$\left.\left.-\frac{2}{105}Q\varepsilon_{11}{}^2\right)\left(\frac{2}{105}Q\varepsilon_{11}{}^2-2\right)+1\right]-\frac{30}{B}R\left(\frac{m\pi}{l}\right)^2\left(\frac{3}{4}-\right.$$
$$\left.-\frac{2}{105}Q\varepsilon_{11}{}^2\right)N_{10}.$$

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According to Hurwitz' criterion, we have the following expression for the prolonged critical force:

$$\sigma_{10}{}^{Д}=\frac{30G}{R^2}\lambda^2\frac{h^2}{12}+\frac{30G}{\lambda^2}\left[\left(2-Q\varepsilon_{11}{}^2\frac{2}{105}\right)-\frac{1}{{}^3/_4-Q\frac{2}{105}\varepsilon_{11}{}^2}\right], \tag{5.11.10}$$

where

$$\lambda=\frac{m\pi R}{l}.$$

After the minimization of this expression on λ, we have

$$\sigma_{10}{}^{D}=30G\frac{h}{R}\sqrt{\frac{1}{3}\left(\frac{1}{2}-\frac{11}{210}Q\varepsilon_{11}{}^2\right)+\frac{4}{11025}Q^2\varepsilon_{11}{}^4}. \tag{5.11.11}$$

Last/latter term under square root as compared with the others is low. If it we reject/throw, let us have

$$\sigma_{10}=\tilde{G}\frac{h}{R}\sqrt{0{,}167-\tilde{Q}\varepsilon_{11}{}^2}, \tag{5.11.12}$$

where

$$\tilde{G}=30G \text{ and } \tilde{Q}=\frac{4}{630}Q.$$

Thus, the prolonged critical force of the compressed cylindrical shell depends on the characteristics of material at the long effective load, accumulated of complete deformation and the geometric parameters of shell.

§5.12. Torsion form of the loss of stability during the nonlinear creep of the material taking into account a difference in the laws of loading and unloading.

The torsion form of loss of stability during creep is accompanied by the combined loading of material. The process of the combined loading of polymeric materials in the case of the long effective axial load with the subsequent torsion is satisfactorily described (see §3.8) by the theory of the locality of deformations, if we consider the difference in the laws of the deformation of material with its loading and unloading. Then the function of the locality of deformations will take the different form depending on that, will shearing stresses on the local pad grow/rise or decreased.

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During accomplishing condition

$$\sigma_{xz} \geqslant \sigma_{xz**} \cos\left(\widehat{\sigma_{xz}, \sigma_{xz**}}\right), \qquad (5.12.1)$$

where σ_{xz} is instantaneous value of shearing stress; σ_{xz**} - any

preceding/previous shearing stress on the pads, the deformation properties of polyethylene with sufficient accuracy/precision, can be described, if we accept the function of the locality of deformations in the following form:

$$\gamma_{xz}=\frac{\sigma_{xz}}{G_{xz}}+\frac{n}{G_{xz}}\dot{\sigma}_{xz}+Q\gamma_{xz}^{\alpha}-nB\dot{\gamma}_{xz}. \qquad (5.12.2)$$

During the unloading when we have the following condition:

$$\sigma_{xz}<\sigma_{xz**}\cos\left(\widehat{\sigma_{xz},\sigma_{xz**}}\right), \qquad (5.12.3)$$

for the satisfactory description of experiments the function of local deformations we take in the form with two at times relaxation:

$$\bar{\gamma}_{xz}=\frac{\sigma_{xz}}{G_{xz}}+\frac{\bar{n}}{G_{xz}}\dot{\sigma}_{xz}+\frac{\bar{\bar{n}}}{G_{xz}}\ddot{\sigma}_{xz}-n\bar{B}_{xz}\dot{\gamma}_{xz}-\bar{n}\bar{B}\ddot{\gamma}_{xz}+Q_{xz}\gamma_{xz}^{\alpha}. \qquad (5.12.4)$$

Theoretical curves with the use of the function of local strains in the form (5.12.2) and (5.12.4) and experimental points during creep with constant stress and reverse/inverse creep after unloading of high-density polyethylene are shown to Fig. 5.4, where the constants of material have the following values:

$$n=10;\quad \bar{n}=40;\quad \bar{\bar{n}}=250;\quad G_{xz}=480;\quad Q_{xz}=8200;$$
$$B_{xz}=\bar{B}_{xz}=\bar{\bar{B}}_{xz}=3{,}5;\quad \bar{Q}_{xz}=820.$$

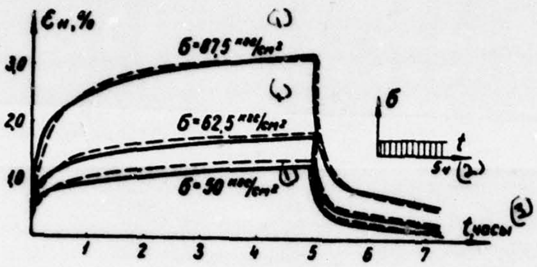

Fig. 5.4. Curves of creep with constant load and during unloading (solid lines is experimental; intermittent - theoretical).

Key: (1). cm². (2). h. (3). hours.

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We utilize the taken frequent forms of the function of the locality of strain for the study of the torsion form of loss of stability during creep. Let us examine thin-walled rod with cruciform section/cut or the narrow band with three hinge-fastened edges, compressed along free long side. Let us assume that the thickness of section/cut is considerably less than its width. Such section/cuts can lose directional stability of twisting. Examining cell/element without initial bending, we see that in subcritical stage the material will be located in the single-axle stressed state. Utilizing the taken above functions of local strains, let us determine

communication/connection between increases in shearing stresses and strains and their velocities during the low deviation (twisting) of cell/element from its subcritical rectilinear form of the equilibrium:

$$\delta\varepsilon_{12}=\frac{1}{S}\int\limits_{s^{(+)}}\delta\gamma_{xz}v_{12xz}ds+\frac{1}{S}\int\limits_{s^{(-)}}\delta\bar{\gamma}_{xz}v_{12xz}ds, \qquad (5.12.5)$$

here $\delta\gamma_{xz}$ is increase of the function of local strains;

v_{12xz} - weighting function (its increase in this task $\delta v_{12xz}\to 0$);

S^+ is a region of the sphere in which is satisfied the condition (5.12.1), and therefore on this region the function of local strains it will take the form according to (5.12.2);

S^- is a region of the sphere in which is satisfied the condition (5.12.3) with the appropriate function of local deformations (5.12.4).

Regions on the sphere in which are satisfied conditions (5.12.1) and with respect (to 5.12.3), with by elephant loading by compression with the subsequent torsion are investigated earlier (§2.9).

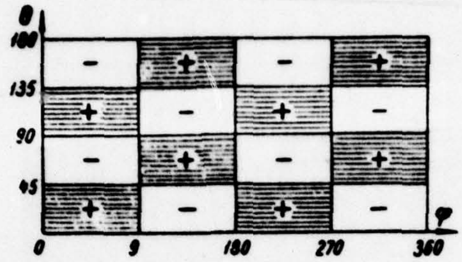

Fig. 5.5. Regions of loading and unloading during the conditional scan/development of sphere.

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During a low increase in shearing stress of condition (5.12.1) and (5.12.3) they divide sphere into two regions, shown to Fig. 5.5, where on the region, designated in plus, is satisfied condition (5.12.1), on remaining region is satisfied the condition (5.12.3).

Integrating (5.12.5) by sphere taking into account of the corresponding to each region form of the function of local strains (5.12.2) and (5.12.4), we come to the relationship/ratio

$$\delta\gamma_{12}=\frac{7}{20}\left(\frac{1}{G_{xz}}\delta\sigma_{12}+\frac{n}{G_{xz}}\delta\dot{\sigma}_{12}-nB_{xz}\delta\dot{\gamma}_{12}\right)+$$

$$+\frac{27}{42}Q_{xz}\cdot\varepsilon_{11\circ}{}^{2}\delta\gamma_{12}+\frac{7}{20}\left(\frac{1}{G_{xz}}\delta\sigma_{12}+\frac{\bar{n}}{G_{xz}}\delta\dot{\sigma}_{12}+\frac{\bar{\bar{n}}}{G_{xz}}\delta\ddot{\sigma}_{12}-\right.$$

$$\left.-\bar{n}\bar{B}_{xz}\delta\dot{\gamma}_{12}-\bar{\bar{n}}\bar{\bar{B}}_{xz}\delta\ddot{\gamma}_{12}\right)+\frac{9}{14}\bar{Q}_{xz}\varepsilon_{11\circ}{}^{2}\delta\gamma_{12} \qquad (5.12.6)$$

or, after equating $B_{xz}=\bar{B}_{xz}=\bar{\bar{B}}_{xz}$ and after designating

$$n_1=\frac{10}{7}(n+\bar{n});\quad G=G_{xz}B_{xz};$$

$$n_2=\frac{10}{7}\bar{\bar{n}};\quad G_1=\frac{7}{20}G_{xz}B_{xz};$$

$$G_{Д}(\varepsilon_{11\circ})=\frac{10}{7}G_{xz}\left[1-\frac{19}{14}(Q_{xz}+\bar{Q}_{xz})\,\varepsilon_{11\circ}{}^{2}\right],$$

we obtain

$$\delta\sigma_{12}+n_1\delta\dot{\sigma}_{12}+n_2\delta\ddot{\sigma}_{12}=$$

$$=G_{Д}(\varepsilon_{11\circ})\delta\gamma_{12}+G_1n_1\delta\dot{\gamma}_{12}+Gn_2\delta\ddot{\gamma}_{12}. \qquad (5.12.7)$$

After the integration of this expression for the cross section of cell/element for free torsion, we have

$$\bar{m}_к+n_1\dot{m}_к+n_2\ddot{m}_к=G_{Д}(\varepsilon_{11\circ})I_к\theta''+G_1n_1\dot{\theta}''+GI_кn_2\ddot{\theta}'', \qquad (5.12.8)$$

where $\bar{m}_к$ is intensity of the external torsional load;

$I_к$- the second moment of area during torsion;

θ is an angle of twist.

During the axial compression of thin-walled cell/element the intensity of the torsional load

$$m_{\kappa}=\sigma_{11}I_p\theta'', \qquad (5.12.9)$$

here I_p is the polar moment of inertia.

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After accepting θ (t,x) = f(t) sin πx/l,

(5.12.10)

for the amplitude of sagging/deflections we have

$$a_2n_2\ddot{f}+a_1n_1\dot{f}+a_0f=0, \qquad (5.12.11)$$

where

$$a_0=G_{Д}(\varepsilon_{11\circ})I_{\kappa}-\sigma_{11}I_p;$$
$$a_1=G_1I_{\kappa}-\sigma_{11}I_p; \quad a_2=GI_{\kappa}-\sigma_{11}I_p.$$

Expression for prolonged critical stress we will obtain, as is known, after equating a_0 with zero:

$$\sigma_{кр}{}^{Д}= G_{Д}(\varepsilon_{11\circ})\frac{I_{\kappa}}{I_p}. \qquad (5.12.12)$$

As is evident, prolonged critical force depends on the prolonged characteristics of material with its loading and the unloading.

accumulated of complete strain (elastic deformation and creep strains) and of the characteristics of the cross section of cell/element (for the section/cuts in question $I_u/I_p = b^2/h^2$, where h are thickness, b - width of cell/element).

The graph of prolonged critical forces with the obtained constants of material is given on Fig. 5.6. For the case in question instantaneous critical force takes value $\sigma_{kp}{}^M = G\frac{h^2}{b^2}$, i.e. more than four times it exceeds prolonged critical force.

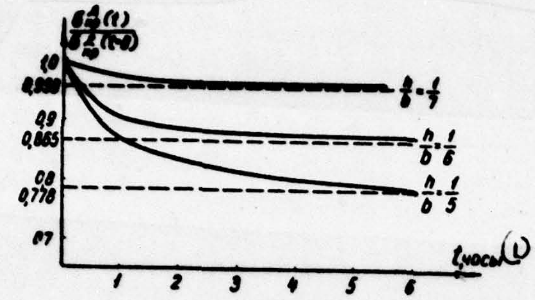

Fig. 5.6. Prolonged critical forces for the different ratios h/b.

Key: (1). hours.

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§5.13. Experimental analysis of stability of cylindrical shells during compression by constant load.

Until now, in the experimental analyses of stability of shells of polymeric materials, was examined mainly the elastic stage of work of material [177-179, 182]; however, stability taking into account prolonged processes was studied insufficiently. In the described below experiments the stability is investigated taking into account the rheonomic properties of material.

The experimental studies of the prolonged stability of cylindrical shells during axial compression were carried out above shells of high-density polyethylene. In all were tested three series, on three specimen/samples in each (Table 5.2).

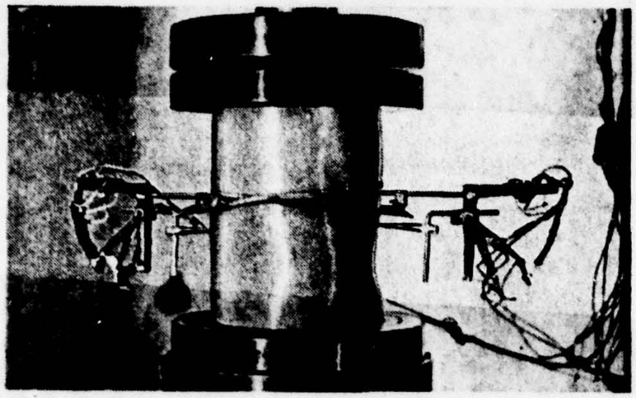

Fig. 5.7. Form of shell after testing of constant load.

Tables 5.2.

Geometric characteristics of experimental models.

(1) Серия	(2) Кол-во образцов	(3) Сред. радиус R_{cp}, мм	(4) Сред. толщина стенки h_{cp}, мм	(5) Сред. площадь поперечного сечения F_{cp}, см²	R/h
I	3	48,7	3,2	10,1	14,8
II	3	48,9	3,7	11,3	13,3
III	3	49,1	4,1	12,6	12,0

Key: (1). Series. (2). Quantity of specimen/samples. (3). Media. a radius R_{cp}, mm. (4). Media. the thickness of wall h_{cp}, of mm. (5). Media. the cross-sectional area F_{cp}, of cm².

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Experimental shells were sharpened on lathe made of thin-walled polyethylene tubes (height/altitude of specimen/samples 13.5 cm; the average value of the modulus of instantaneous elasticity H = 7000 kg/cm²).

Specimens were instantly charged by the constant, long effective load. At the initial torque/moment after the load of bulges in specimen/sample, it was not observed. In the course of time in specimen/sample, began to be formed annular bulges (Fig. 5.7). Was fixed the torque/moment of the time when in specimen/sample appeared the first, clearly distinguishable annular bulge.

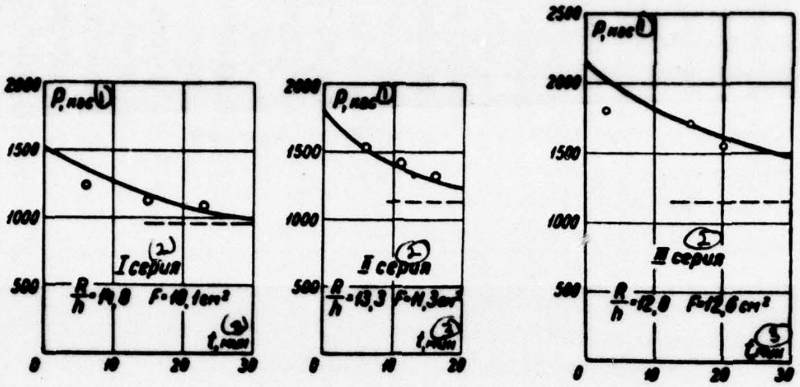

Fig. 5.8. The theoretical curve of prolonged critical force according to expression (5.13.1) and experimental points (o) ($\tilde{G}$ = 6000; $\tilde{Q}$ = 85).

Tables 5.3.

Results of experiments.

(1) Серия	(2) № образца	(3) Напряжение σ, кг/см²	(4) Время до появления первой выпучины, мин	(5) Замеренная на образце полная деформация $\varepsilon_{11}\cdot 10^3$
I	1	109	23	3,57
	2	114	15	3,10
	3	124	6	2,76
II	1	118	16	3,48
	2	126	11	3,31
	3	135	6	3,28
III	1	123	20	3,54
	2	134	15	3,50
	3	142	3	[illegible]

Key: (1). Series. (2). No specimen/sample. (3). Stress σ, kg/cm². (4). Time to the appearance of first bulge, min. (5). Measured in

specimen/sample complete strain $\varepsilon_{11} \cdot 10^2$.

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Test results are given in table 5.3 and are described by dependence (5.11.12) V. Prager's tank kinematic model. The law of the development of local strains is accepted both in final and differentially. In the first case the way of loading is considered only by the regions of additional charge and unloadings, in the second - also by integral in terms of the local way of loading.

The obtained theoretical expressions of strains satisfactorily describe the experimental results of the combined loading of the elasto-plastic strengthened/hardened material, which confirms the correctness of the initial prerequisite/premises of theory.

The rheonomic properties of material, according to the theory of the locality of deformations, are described by means the examination of the function of the locality of deformations as of time/temporary dependence. In connection with polymeric materials most promising is the use of dependences of Bol'tsmana - Volterra's hereditary theory. Since experiments establish/installed that for many polymeric materials the laws of deformation with loading differ from law during

unloading of material, for the solution of the problems of combined loading is establish/installed the criterion of using the corresponding laws. Are obtained the relationship/ratios between strains and stresses with the simple and combined loadings of nonlinear creeping material, if loads change steplike.

The results of creep tests (in nonlinear range) of high-density polyethylene (PVP) with the simple and complex steplike ways of loading confirm the prerequisite/premises of theoretical relationship/ratios.

The obtained components of the tensor of compliance/pliability for the torque/moment of the bulge of shells in moment-less subcritical state show their essential dependence on the way of subcritical loading. The level of the critical stressed state will be minimum with the simple subcritical loading when additional plastic deformations grow/rise in all local coordinate systems. The maximum level of breaking stresses is obtained with this complex way of the loading when do not appear additional plastic deformations. Minimum breaking stresses, received by the theory of the locality of deformations, close to the results of deformation theory.

The carried out tests above thin-walled cylindrical shells of polyethylene with short-term loading showed satisfactory agreements

of theoretically calculated critical forces with the results of experiment. The zone of experimental breaking stresses lie/rests at the zone between the values of upper and lower theoretical breaking stresses.

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With an increase in the plastic deformations, the effect of geometric nonlinearity is smoothed. The obtained theoretical stresses will agree also with the results of the experience of A. S. Vol'mir, A. N. Vozhinskiy and V. S. Moskvitin on the analysis of stability of thin-walled rods, plates and shells of alloy D-16T.

Those who were solved on the base of the refined kinematic model of the task of the prolonged stability of plates and shells of reinforced plastic showed that the disregard of transverse shift/shears in the large ratio E/G leads to significant quantitative error. In a number of cases when creep in the direction of reinforcement can be disregarded as compared with shift of creep, classical kinematic model even it is qualitative not in state to describe the bulge of plates and shells in time. The solved particular tasks of the stability of plates and shells during the creep of the material in geometrically linear and nonlinear settings (during linear creep) made it possible to establish/install the

limits of the applicability of Kirchoff - Love's kinematic model.

Is obtained the load of the prolonged stability of shells of nonlinear-creeping, initial-isotropic material for two cases: when reverse/inverse creep is described by the law of creep with stressing and when reverse/inverse creep is described by the law, which differs from law so on loading.

The carried out tests of cylindrical shells of polyethylene by constant load made it possible to fix time when in specimen/sample begin to appear the first bulges.

The results of experiments are satisfactorily described by the dependences, obtained theoretically.

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